

Effect of Environmental Temperature on the Lower Bound of the Entropic Uncertainty Relation in the Presence of Quantum Memory

Fazlahmad Frozan^{1*}, Abdul Qauom Shadab², Awaz Bromand³

^{1,2,3} Department of Physics Ghor University

ABSTRACT: Because realistic quantum systems inevitably interact with their surroundings, understanding how environmental factors affect the entropic uncertainty relation remains a major challenge. In this work, we analyze how ambient temperature impacts the lower bound of the memory-assisted entropic uncertainty relation. We model the coupling between the system and its thermal environment using a generalized amplitude damping channel, featuring temperature-dependent parameters. Notably, we find that the uncertainty lower bound drops when the damping parameter (r) decreases, and rises when it increases. By contrast, shifting the parameter (p) yields much smaller variations. This stark difference highlights our main novel finding: in a finite-temperature environment, the lower bound of the entropic uncertainty relation is significantly more sensitive to the damping parameter (r) than to (p). Ultimately, these results offer a clearer look at how thermal reservoirs influence open quantum systems and affect practical quantum information processing.

KEYWORDS: quantum memory, the lower of entropic uncertainty relation, the effect of environment temperature

I. INTRODUCTION

The history of quantum information began in the early twentieth century, when classical physics became quantum physics. Quantum mechanics was formulated by Schrödinger using wave mechanics and Heisenberg using matrix mechanics [1]. The equivalence of these methods later proved to be that their formulas described the dynamics of microscopic systems but had several undesirable aspects in describing measurement processes. Von Neumann then developed quantum theory using operator algebra to describe measurement, as well as dynamics [2]. These studies emphasized the philosophical aspects of measurement to express a quantitative approach to information extraction through measurement. In the 1960s, Stratonovich, Helstrom, and Gordon proposed a formula for optical communication using quantum mechanics [3]. This was the first historical emergence of quantum information theory. They mainly studied the probability of error and the capacity of the channel to communicate. Later, Holevo achieved a high level of communication speed in transmitting a classical message through a quantum channel. At a time when computer science was revolutionizing, information and communication theory also evolved through Claude Shannon[5] Shannon developed two basic theories of information theory: the no-noise channel coding theorem and the noise-free channel coding theorem. He also showed that error correction codes can be used to protect the information sent. Quantum information theory followed a similar path, with Ben Schumacher simulating Shannon's noise-free coding theorem in 1995 using qubits. Error correction theory was also developed, which allows quantum computers to perform efficient computations without regard to noise and to communicate securely over noisy quantum channels. Then the interaction of quantum radiation with various particles in the last decade has caused great scientific interest. Quantum memory is one of these fields that draws the quantum state of light on a group of atoms and then restores it to its original form. Quantum memory is a key element in information processing, such as optical quantum computing and quantum communication, while opening up new avenues for the interaction of light and atoms. However, restoring the quantum state of light is not an easy task. While significant progress has been made, researchers are still working to make it happen [6].

Quantum memory based on quantum exchange allows the storage of photon qubits. Kessel and Moiseev discussed quantum storage in single photon mode in 1993. The experiment was analyzed in 1998 and demonstrated in 2003. In the single photon mode, it can be considered a product of the classic optical data storage technology introduced in 1979 and 1982. Not only that, but the idea was inspired by the high density of data storage in the mid-1970s. Quantum information can be transmitted by photons. Where

atoms or host matter are used as information stores that acts as quantum memories during the time a photon is trapped, quantum memories generally represent a mixed or pure state.

The uncertainty principle was first proposed by Heisenberg [7]. It clearly shows the difference between classical and quantum mechanics. This principle limits the uncertainty about the results of two incompatible measurements, such as the location and momentum in a particle. In this dissertation, we extend the uncertainty principle to the case where a particle is intertwined with quantum memory, and offer a lower range for uncertainties, which depends on the degree of entanglement between the particle and quantum memory.

In this article, we will first examine the basic concepts of classical and quantum information and then introduce the reasons for the importance of quantum information to justify many physical phenomena. We then studied the relationships between the uncertainty principle and quantum correlations. We also show the relationships between the principle of uncertainty in the presence of quantum memory and its applications. Finally, we examine the results of the effect of ambient temperature on the lower limit of the relationship of uncertainty in terms of entropy in the presence of quantum memory.

Ambient temperature at the lower limit investigates the relationship of uncertainty in terms of entropy in the presence of quantum memory.

The novelty of this work lies in investigating the effect of environmental temperature on the lower bound of the entropic uncertainty relation in the presence of quantum memory under a generalized amplitude damping channel. Furthermore, we quantitatively compare the influence of the environmental parameters p and r and show that the uncertainty lower bound is more sensitive to variations of parameter r than parameter p .

A. Research Method

Using the resources available in the library and using the available articles, we study the relationship of uncertainty in terms of entropy in the presence of quantum memory and then the effect of ambient temperature on the boundary of the relationship of entropy uncertainty in the presence of quantum memory. Finally, we perform analysis and calculations and compare the results with others. The hypothesis in this research we focus in tow section:

1. Quantum system interacts with its surroundings.
2. The environment has several effects on quantum correlation

B. Research Question

- 1- Will the ambient temperature have an effect on the uncertainty relationship?
- 2- Does the ambient temperature increase the uncertainty relationship?

II. UNCERTAINTY PRINCIPLE

Uncertainty Principle. The uncertainty principle was first coined in 1927 by Werner Heisenberg using the Max Planck hypothesis. Heisenberg formulated the principle of uncertainty while working on the mathematical foundations of quantum mechanics at the Niels Bohr Institute in Copenhagen, Denmark. Hence, it is safe to say that the principle of uncertainty is one of the concepts that distinguishes quantum mechanics from classical theories. According to this principle, precise values can never be attributed simultaneously to the location and motion of a physical system. In fact, the word uncertainty has several meanings in terms of physical concepts, among which we can mention these cases. As the inaccuracy in measuring a physical quantity, as the lack of knowledge and awareness of the observer of the desired value, and in other cases the statistical dispersion in a set whose components are systems that are similar in every way and to measure a quantity are taken[8].

C. Heisenberg uncertainty principle.

Heisenberg's uncertainty principle began with a simple reasoning. According to Heisenberg, there is no physical phenomenon in itself unless it occurs between two observers or two objects. In classical mechanics, the object and its path are determined based on the Euclidean geometry of the object, and thus the path of the object is determined by a mathematical point and line of explanation. But according to Heisenberg, this assumption is true in classical mechanics, while in quantum mechanics the situation can be different and change. Whenever we want to evaluate the position of a particle like a photon with an electron, we have to have a small effect on its path. We can only examine this quantity carefully if we do not reveal its nature and change its structure, which would seem impossible with today's facilities and tools. Thus, Heisenberg showed by his equations that if we made such a device

in our minds, we could not make the desired measurement with the desired accuracy. On the other hand, the equations of Heisenberg's uncertainty principle limit any variable and measurable quantity. As we know, particles are described according to the complementary principle, that is, a particle shows both a wave state and a particle state. That is, when the property of a wave is considered, the quantum-related quantities are measured and the particle quantities are measured indefinitely, and if we want to consider the uncertainty principle for the particle state, the zero-particle properties and the wave-related quantities will be infinitely measured. Heisenberg's uncertainty principle in quantum physics states that certain pairs of physical properties, such as location and magnitude of motion (momentum), cannot be determined with optional accuracy. In other words, increasing accuracy in quantity is one of the characteristics of location or size of motion, while decreasing accuracy in quantity is another characteristic. Hence, every quantum state has the characteristic that the product of the mean square root of the deviations from the mean value of space, at the root of the mean square of the deviations from the mean value of the motion (momentum), can never be less than a constant ratio of Planck constant. In other words.

$$\begin{cases} \Delta x = \sqrt{\langle (x - \langle x \rangle)^2 \rangle} \\ \Delta p = \sqrt{\langle (p - \langle p \rangle)^2 \rangle} \end{cases} \Rightarrow \Delta x \Delta p \geq \frac{\hbar}{2} \quad (1)$$

One of these cases is the study of the effects of the environment on this relationship. Since in the real world it is not possible to separate quantum systems from the environment and therefore the system is interacting with the environment, the study of the effects of the environment on the relationship of uncertainty in terms of entropy is of particular importance. In this dissertation, we try to investigate the effect of ambient temperature on the lower limit of the relationship of uncertainty in terms of entropy in the presence of quantum memory.

D. Environmental Noises

In this paper, zero temperature refers to absolute zero temperature ($T = 0$ K). Temperature is not introduced directly into the uncertainty relation itself; rather, it affects the quantum system through its interaction with a thermal reservoir. The temperature determines the probabilities of excitation and relaxation processes and therefore influences the evolution of the density matrix and the resulting entropic uncertainty lower bound.

In the real-world, quantum systems interact with their environment, so the study of the effects of the environment on the system from various aspects has been the subject of much research. One of these important topics is the study of the effects of the environment on quantum correlations, including entanglement in quantum systems. Because quantum entanglement plays an important role in entropy uncertainty in the presence of quantum memory. Investigating the effects of the environment on the lower limit of the uncertainty relation can be interesting when a quantum system interacts with its environment at zero temperature, where $\hbar\omega \gg k_B T$, where $\hbar\omega$ is the transmission energy of the quantum system and k_B is Boltzmann. Its temporal evolution can be represented using a domain damping channel, which is defined as follows[9].

$$\varepsilon_{AD}(\rho) = \sum_{i=0}^1 E_i \rho E_i^\dagger \quad (2)$$

As we have

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-r} \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0 & \sqrt{r} \\ 0 & 0 \end{bmatrix}$$

Where E_0 and E_1 are the Krauss operators, and r the probability of a transition from the excited state $|1\rangle$ to the base state $|0\rangle$ is zero. It should be noted here that the only possible transition is from a higher energy state to a lower energy state. It should be noted here that in the real world, it is impossible to have a zero temperature environment. When the ambient temperature is non-zero, the situation is completely different from the previous case, because in this case, in addition to losing excitation, the quantum system can be excited as a result of interaction with the environment. Such an interaction can be demonstrated using a generalized amplitude attenuation channel, as follows [9].

$$\begin{aligned}\varepsilon_{\text{GAD}}(\rho) &= \sum_{i=0}^3 E_i \rho E_i^\dagger \\ E_0 &= \sqrt{p} \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-r} \end{bmatrix} \\ E_1 &= \sqrt{p} \begin{bmatrix} 0 & \sqrt{r} \\ 0 & 0 \end{bmatrix} \\ E_2 &= \sqrt{1-p} \begin{bmatrix} \sqrt{1-r} & 0 \\ 0 & 1 \end{bmatrix} \\ E_3 &= \sqrt{1-p} \begin{bmatrix} 0 & 0 \\ \sqrt{r} & 0 \end{bmatrix}\end{aligned}$$

The damping channel describes the amplitude of a situation in which the system can both damage and excite by interacting with the environment. In certain physical systems, r can be related to the amount of energy relaxation, and $\{p, r\}$ is usually a function of temperature. Here we want to get the relationship between channel parameters (p, r) and the physical properties of the system and the environment [10]. Consider a two-level atom in a thermal bath. Each qubit state can be represented by the Bloch butter vector as follows:

$$\rho = \frac{1}{2} (I + \mathbf{k} \cdot \boldsymbol{\sigma}) \quad \mathbf{k} = (k_1, k_2, k_3), \quad \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)^T$$

Where $\sigma_1, \sigma_2, \sigma_3$ are Pauli matrices?

After the effect of an extended domain damping channel, the output state is as follows:

Is $\tilde{\rho} = \varepsilon_{\text{GAD}}(\rho)$, we have $\tilde{\rho} = \frac{1}{2} (I + \tilde{\mathbf{k}} \cdot \boldsymbol{\sigma})$ which

$$\tilde{\mathbf{k}} = [k_1 \sqrt{1-r}, k_2 \sqrt{1-r}, k_3(1-r) + r(2p-1)]$$

For a two-level system in a thermal bath with temperature T , the basic equation will be as follows:

$$\frac{d\rho(t)}{dt} = \gamma_0(N+1) \left[\sigma_- \rho(t) \sigma_+ - \frac{1}{2} \sigma_+ \sigma_- \rho(t) - \frac{1}{2} \rho(t) \sigma_+ \sigma_- \right] + \gamma_0 N \left[\sigma_+ \rho(t) \sigma_- - \frac{1}{2} \sigma_- \sigma_+ \rho(t) - \frac{1}{2} \rho(t) \sigma_- \sigma_+ \right]$$

Where γ_0 is the spontaneous emission rate of the atom, $\sigma_- \sigma_+$ (atomic increasing (decreasing) operators and

$N = \left[\exp\left(-\frac{\hbar\omega}{k_B T}\right) - 1 \right]^{-1}$. We can also write $\rho(t)$ as $\rho(t) = \frac{1}{2} [I + \mathbf{k}(t) \cdot \boldsymbol{\sigma}]$, then the basic equation $\mathbf{k}(t)$ is as follows:

$$-\frac{\gamma}{2} k_2, -\gamma k_3 + \gamma_0 \frac{d\mathbf{k}(t)}{dt} = \left(-\frac{\gamma}{2} k_1\right.$$

Where $\gamma = \gamma_0(2N+1)$ and we have.

$$\mathbf{k}(t) = \left(k_1 e^{-\frac{\gamma}{2}t}, k_2 e^{-\frac{\gamma}{2}t}, \left(k_3 - \frac{\gamma_0}{\gamma} \right) e^{-\gamma t} + \frac{\gamma_0}{\gamma} \right)$$

We assume that $\tilde{\mathbf{k}} = \mathbf{k}(t)$ so we have:

$$r = (1 - e^{-\gamma t})$$

$$p = \frac{1}{1 + e^{\frac{\hbar\omega}{k_B T}}} \quad (3)$$

To establish the relationship between the generalized amplitude damping channel parameters and the physical properties of the environment, we consider a two-level atom interacting with a thermal reservoir at temperature T . The dynamics of the system can be described within the Born–Markov approximation using the Lindblad master equation:

$$\frac{d\rho}{dt} = \gamma(N+1) \left(\sigma_- \rho \sigma_+ - \frac{1}{2} \{ \sigma_+ \sigma_-, \rho \} \right) + \gamma N \left(\sigma_+ \rho \sigma_- - \frac{1}{2} \{ \sigma_- \sigma_+, \rho \} \right),$$

where ρ is the density matrix of the atom, γ is the spontaneous emission rate, N is the mean thermal photon number of the reservoir, and σ_+ and σ_- are the atomic raising and lowering operators, respectively.

Considering the excited-state population

$$p(t) = \rho_{ee}(t),$$

and taking the matrix element corresponding to the excited state, we obtain

$$\frac{dp(t)}{dt} = \gamma(2N+1)p(t) + \gamma N.$$

Solving this first-order differential equation yields

$$p(t) = p_{\infty} + [p(0) - p_{\infty}]e^{-\gamma(2N+1)t},$$

Where

$$p_{\infty} = \frac{N}{2N+1}$$

is the thermal equilibrium population. This result shows explicitly how the environmental temperature enters the system dynamics through the thermal occupation number N , which is related to temperature by the Bose–Einstein distribution

$$N = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}.$$

Therefore, the generalized amplitude damping parameter p is temperature-dependent and characterizes the influence of the thermal environment on the quantum system.

As we see, the generalized amplitude damping parameter p depends on the ambient temperature. Note that the equation is reduced to Equation by setting $p = 1$. In the generalized amplitude attenuation channel, an atom can not only be transferred from the excited state to the ground state by spontaneous emission, but can also move from the ground state to the excited state by absorbing energy from the environment at a finite temperature[10].

III. INVESTIGATION OF THE EFFECT OF ENVIRONMENTAL PARAMETERS ON THE LOWER LIMIT OF UNCERTAINTY

Here we consider a two-part quantum AB system, in which each individual part interacts independently with the environment at a non-zero temperature. The evolution of this quantum system in Krauss representation is given by the following equation.

$$\rho_{AB}(t) = \sum_{i,j=0}^3 (E_i \otimes E_j) \rho_{AB}(0) (E_i \otimes E_j)^{\dagger},$$

Here $\rho_{AB}(0)$ is the initial state of the two-qubit system. In the next step, Alice makes a measurement on her system. The state of the system after measurement will be as follows.

$$\rho_{OB} = \sum_i (|o_i\rangle\langle o_i| \otimes I) \rho_{AB}^{(t)} (|o_i\rangle\langle o_i| \otimes I),$$

Where we $\{|o_i\rangle\}$ are specific to the observable vectors o and I is the same operator as subsystem B. Assume that the initial state of the Werner system is defined as follows[12].

$$\rho_w = \frac{1-q}{4} I + q |\varphi^+\rangle\langle\varphi^+|$$

Where $|\varphi^+\rangle$ is equal to:

$$|\varphi^+\rangle = \frac{1}{\sqrt{2}} [|0,0\rangle + |1,1\rangle]$$

A maximum state is intertwined. Now we want to get the effects of a non-zero temperature environment on the lower limit of the uncertainty relation for this case. Here we consider the lower bound of the Bertha uncertainty relation, whose lower bound is:

$$\log_2 \frac{1}{c} + H(A|B)$$

Here we assume that Alice measures one of the observations σ_x and σ_z on his subsystem, resulting in $\log_2 \frac{1}{c} = 1$

In the following figures, we have plotted the uncertainty relation for the various parameters of the lower boundary environment (p, r). In Figures 1 to 5, the parameters of the environment AB are considered the same, ie $r_A = r_B$ and $p_A = p_B$.

In all shapes, the dashed graph is black for the case $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$.

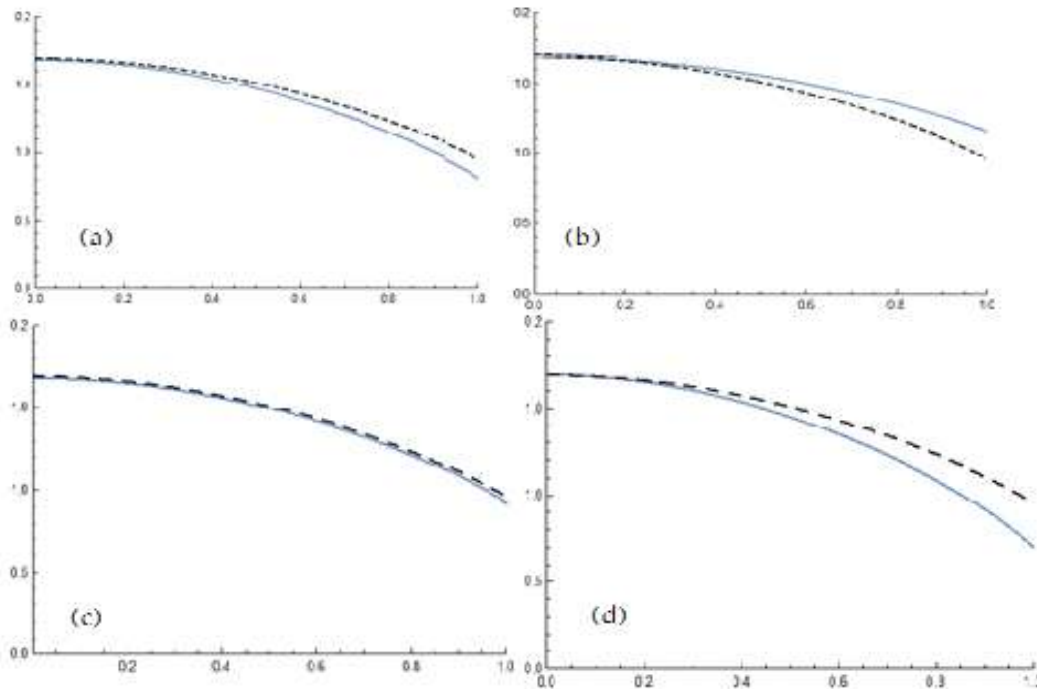


Fig. 1. The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system

The figure 1a shows that the lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system[13], and it is assumed that the observations σ_x and σ_z are measured on subsystem A. Here is the black dashed line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$ and the blue line for the case where $p_A = p_B = 0.1$ and $r_A = r_B = 0.1$.

The figure 1b shows that the lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system, and it is assumed that the observations σ_x and σ_z are measured on subsystem A. Here is the black dashed line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$ and the blue line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.3$.

The Figure 1c The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system, and it is assumed that the observations σ_x and σ_z are measured on subsystem A. Here is the black dashed line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$ and the blue line for the case where $p_A = 0.9, p_B = 0.4$ and $r_A = r_B = 0.2$.

Figure 1d The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner mode system, and it is assumed that the observations of σ_x and σ_z are measured on subsystem A. Here is the black dashed line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$ and the blue line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.1$ [16].

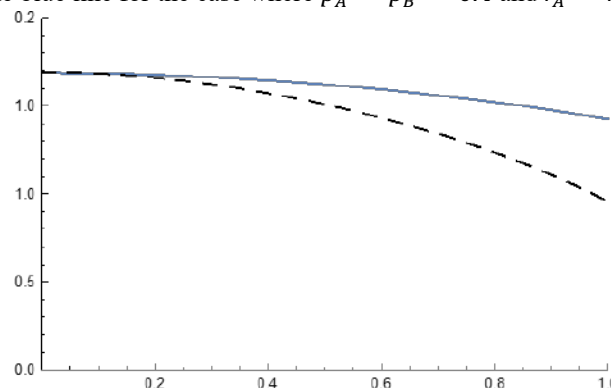


Fig. 2. The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system

Figure 2 The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system, and it is assumed that the observations σ_x and σ_z are measured on subsystem A. Here is the black dashed line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$ and the blue line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.5$.

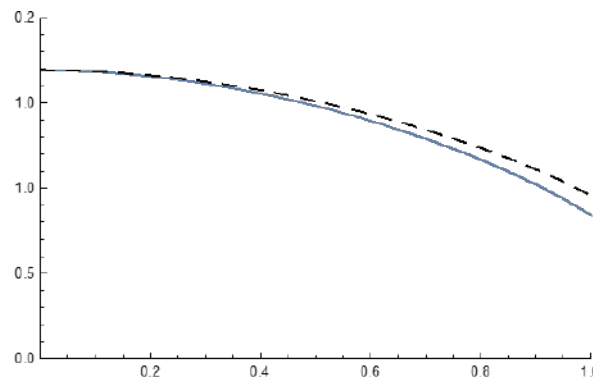


Fig. 3. The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system

Figure 3 The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system, and it is assumed that the observations σ_x and σ_z are measured on subsystem A. Here is the black dashed line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$ and the blue line for the case where $p_A = p_B = 0.4$ and $r_A = 0.2, r_B = 0.1$.

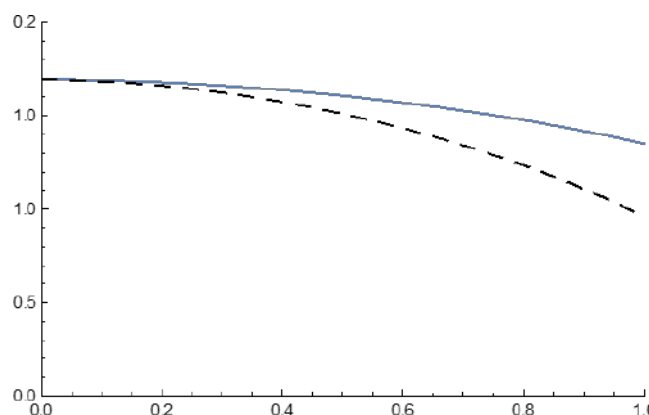


Fig. 4. The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system

Figure 4 The lower bound of the uncertainty relation in terms of q is in the initial state of the Werner state system, and it is assumed that the observations σ_x and σ_z are measured on subsystem A. Here is the black dashed line for the case where $p_A = p_B = 0.4$ and $r_A = r_B = 0.2$ and the blue line for the case where $p_A = p_B = 0.4$ and $r_A = 0.2, r_B = 0.6$. [17].

As we can see in Figure 1, decreasing the lower boundary p and r decreases the uncertainty, and Figures 2 and 3 show that increasing the lower boundary p and r increases the uncertainty.

Next, we want to examine which of the environmental parameters plays a greater role in the lower bound of the uncertainty relationship [18].

In Figures 1 to 4 of the lower bound, we draw the uncertainty relation for the parameters ($p_A = p_B$) and the different r s, as can be seen that the lower bound of the uncertainty relation changes with the change of the r parameters [19].

In this way, with increasing r of the lower boundary environment, the uncertainty relationship increases, as can be seen in the figures.

Here it can be concluded that the lower bound of the uncertainty relationship depends on the parameters of the environment. However, the extent of this dependence can vary for different parameters [20].

The results presented in Figs. 1(b)–1(d) and Figs. 2–4 indicate that the uncertainty lower bound exhibits a stronger dependence on parameter r than on parameter p , when the effect of each parameter is examined separately while keeping the other parameter fixed.

IV. CONCLUSION

Conclusion For quantum systems, there are features that are not found in classical systems. One of these features is the entanglement between quantum systems. Entanglement as a key factor plays a significant role in quantum computing and information processing. Even some intertwined states are useful in quantum computing. One of the finite quantum computational algorithms and remote preparation is one of the non-interlaced states that has been found to be useful. Determining the properties and extent of quantum correlations is important both theoretically and practically. Extensive research has been done to understand quantum correlations, as well as various criteria for measuring these correlations. One of these criteria is quantum mismatch. Quantum mismatch determines quantum correlation, which also includes entanglement. In pure conditions, quantum entanglement is equal to pure states of quantum dissonance. But for the most part, mixed states, although not intertwined, have quantum inconsistencies for them. Many of these states are difficult to calculate in practice. Only for dual qubit systems can the discrepancy be calculated. For this reason, they defined other criteria for determining the amount of quantum correlation. A criterion whose basis is defined in terms of the disproportion of the operators and the density matrix, as well as on the basis of reciprocal heterogeneity. The reciprocal heterogeneous bases are also essentially specific to complementary operator vectors.

According to Heisenberg's uncertainty principle, it is not possible to measure both observable with each other, that is, having one system will reduce the uncertainty compared to measuring the other. The basis of the formula is uncertainty in the presence of quantum memory, which for the lower limit expresses the measurement uncertainty of two complementary operators despite the correlation. In this dissertation, we calculated the relationship of ambient temperature uncertainty for the lower bound. We have shown the environmental factors influencing this quantity.

We observed that the lower bound of the uncertainty relation does not change much with the change of the p parameter and it can be said that the lower bound of the uncertainty relation is almost the same for different p s.

We also show that the lower bound of the uncertainty relationship changes with the change of r parameters in such a way that with an increasing r of the lower boundary environment, the uncertainty relation increases. Here, it can be concluded that the lower bound of the uncertainty relationship depends on the parameters of the environment. However, the extent of this dependence can vary for different parameters.

For example, we showed that the lower bound dependence of the uncertainty relationship in the generalized amplitude myra channel to the r parameter is greater than its dependence on the p -parameter of the environment.

In this work, we investigated the effect of environmental temperature on the lower bound of the entropic uncertainty relation in the presence of quantum memory. The thermal environment was modeled using the generalized amplitude damping channel. Our analysis demonstrates that the uncertainty lower bound depends on the environmental parameters p and r . However, the dependence on r is considerably stronger than the dependence on p . The results show that increasing r increases the uncertainty lower bound, whereas variations in p lead to relatively minor changes. Therefore, the damping parameter r plays the dominant role in determining the behavior of the entropic uncertainty relation under thermal environmental effects. These findings contribute to a better understanding of environmental influences on quantum information protocols and quantum memory systems.

Our numerical analysis shows that the uncertainty lower bound depends on both environmental parameters p and r . However, when the parameters are examined separately, the lower bound exhibits a stronger sensitivity to variations in r than in p .

REFERENCES

1. G.D. Mahan, Quantum mechanics in a nutshell, Princeton University Press, 2008.
2. H.S. PERLMAN, Equivalence of the Schroedinger and Heisenberg Pictures, Nature. 204 (1964) 771–772. <https://doi.org/10.1038/204771b0>.
3. J. Von Neumann, Mathematical foundations of quantum mechanics, Princeton university press, 2018.
4. J.P. Gordon, Quantum effects in communications systems, Proc. IRE. 50 (1962) 1898–1908.
5. C.W. Helstrom, Quantum detection and estimation theory, J. Stat. Phys. 1 (1969) 231–252.
6. C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27 (1948) 379–423.

7. C.E. Shannon, W. Weaver, The mathematical theory of communication. Univ, (1949).
8. J.-L. Le Gouët, S. Moiseev, Quantum memory, J. Phys. B At. Mol. Opt. Phys. 45 (2012) 120201. <https://doi.org/10.1088/0953-4075/45/12/120201>.
9. N. Ohlsson, S. Kröll, S.A. Moiseev, Delayed single-photon self-interference—A double slit experiment in the time domain, in: Coherence Quantum Opt. VIII, Springer, 2003: pp. 383–384.
10. M.A. Nielsen, I.L. Chuang, Quantum computing and quantum information, (2000).
11. S. Barnett, Quantum information, Oxford University Press, 2009.
12. J. Mateo, Efficient information reconciliation for quantum key distribution, Ph. D. thesis, Universidad Politecnica de Madrid, 2011.
13. T.M. Cover, Elements of information theory, John Wiley & Sons, 1999.
14. Č. Brukner, A. Zeilinger, Conceptual inadequacy of the Shannon information in quantum measurements, Phys. Rev. A. 63 (2001) 22113.
15. C. Weedbrook, S. Pirandola, R. García-Patrón, N.J. Cerf, T.C. Ralph, J.H. Shapiro, S. Lloyd, Gaussian quantum information, Rev. Mod. Phys. 84 (2012) 621.
16. D. Petz, M. Rédei, John von Neumann and the theory of operator algebras, World Scientific Singapore, 1995.
17. W. Heisenberg, Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik, in: Orig. Sci. Pap. Wissenschaftliche Orig., Springer, 1985: pp. 478–504.
18. J.J. Sakurai, J. Napolitano, Modern Quantum Mechanics. 2-nd edition, Pers. New Int. Ed. (2014).
19. H.P. Robertson, The uncertainty principle, Phys. Rev. 34 (1929) 163.
20. I. Białynicki-Birula, J. Mycielski, Uncertainty relations for information entropy in wave mechanics, Commun. Math. Phys. 44 (1975) 129–132.

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