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Comparative Study of the Liquid Drop and Collective Models for Nuclear Binding and Low-Energy Structural Properties of Selected Nuclei

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ABSTRACT: This study comparatively evaluates the applicability of the liquid drop and collective nuclear models in describing the energy and structural properties of four selected even–even nuclei, ^{152}Sm, ^{168}Er, ^{132}Sn, and ^{208}Pb. The nuclei were deliberately selected to represent two contrasting structural regimes: deformed, collective systems and nearly spherical, closed-shell systems. The study is a review-based comparative analysis supported by numerical evaluation and validation against published nuclear data. For the liquid-drop description, nuclear binding is considered through the semi-empirical mass-formula framework, including volume, surface, Coulomb, asymmetry, and pairing contributions. The collective description is assessed through low-lying excitation energies, rotational-band systematics, the ratio $R_{4/2} = E(4_1^+)/E(2_1^+)$, and the quadrupole deformation parameter β_2. Binding-energy information was taken from the AME2020 evaluation, while level-structure information was drawn from ENSDF/NNDC and IAEA nuclear-data resources. The analyzed values give B/A of 8.24, 8.13, 8.36, and 7.87 MeV for ^{152}Sm, ^{168}Er, ^{132}Sn, and ^{208}Pb, respectively. The corresponding $E(2_1^+)$ values are 121.8, 79.8, 4041, and 4085 keV, while β_2 is 0.30, 0.34, approximately zero, and approximately zero. The results show that the liquid-drop framework is effective for global binding-energy trends, whereas the collective model provides a more appropriate description of low-energy rotational behavior and deformation in ^{152}Sm and ^{168}Er. Neither framework alone describes all observed features; closed-shell effects require explicit microscopic treatment.

KEYWORDS: collective model, liquid drop model, nuclear binding energy, nuclear deformation, rotational bands.

INTRODUCTION

Nuclear structure is a many-body quantum problem in which protons and neutrons interact through the strong nuclear interaction, electromagnetic effects, pairing correlations, and shell structure. A complete microscopic solution is generally impractical for medium and heavy nuclei, and effective nuclear models therefore remain essential for interpreting measured observables. Different models emphasize different aspects of the same physical system rather than providing universally equivalent descriptions [1,2].

The liquid-drop model provides a macroscopic description in which the nucleus is treated as a nearly incompressible, charged nuclear fluid. Within this picture, the dominant contributions to the binding energy arise from volume, surface, Coulomb, asymmetry, and pairing terms. The resulting semi-empirical mass-formula framework is successful in describing broad mass and binding-energy systematics, but it does not explicitly reproduce shell closures or the detailed low-lying excitation spectrum [3,1].

The collective model complements the macroscopic description by allowing coherent nuclear degrees of freedom, particularly quadrupole vibrations and rotations. In deformed nuclei, the low-lying spectrum can organize into rotational bands, and observables such as $E(2_1^+)$, $E(4_1^+)$, $R_{4/2}$, and β_2 provide practical indicators of collectivity and deformation [4,5].

The thesis on which this article is based compares ^{152}Sm, ^{168}Er, ^{132}Sn, and ^{208}Pb. The first two nuclei are treated as deformed systems with pronounced collective behavior, whereas ^{132}Sn and ^{208}Pb represent closed-shell or doubly magic systems with approximately spherical structure. This selection permits comparison across two structurally distinct regimes and allows the relationship between binding, deformation, excitation energy, and shell effects to be examined.

The objective is to assess the domains in which the liquid-drop and collective models provide useful descriptions of the selected nuclei. Specifically, the study compares binding-energy systematics, low-lying excitation energies, rotational ratios, deformation parameters, and basic electromagnetic characteristics, and evaluates whether model performance depends on both the nuclear property considered and the underlying structural regime.

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MATERIALS AND METHODS

Study design and selected nuclei

This work is a review-based comparative nuclear-structure study. No new irradiation, spectroscopy, or other direct nuclear experiment was performed. The analysis combines theoretical model formulations, numerical evaluation reported in the thesis, and published evaluated nuclear data. The four nuclei were selected to span two contrasting regimes: ^{152}Sm and ^{168}Er as deformed/collective nuclei, and ^{132}Sn and ^{208}Pb as closed-shell or doubly magic, approximately spherical nuclei.

Theoretical framework

For the liquid-drop description, the binding energy is represented within the semi-empirical mass-formula framework as the sum of the principal macroscopic contributions: volume, surface, Coulomb, asymmetry, and pairing terms. In compact form, $B(A,Z) = B_v - B_s - B_C - B_a + \delta$, where the terms represent the volume, surface, Coulomb, asymmetry, and pairing contributions, respectively. The mean binding energy per nucleon is B/A. This framework is used to describe global binding-energy behavior rather than detailed shell-dependent spectra.

For the collective description, low-lying spectra are examined through coherent rotational and vibrational degrees of freedom. For an ideal axially symmetric rotational band, the level energies have the characteristic dependence $E(I) \propto I(I+1)$, while $R_{4/2} = E(4_1^+)/E(2_1^+)$ is used as a structural diagnostic. The thesis also uses β_2 as an indicator of quadrupole deformation. Large β_2, low $E(2_1^+)$, and a relatively large $R_{4/2}$ are interpreted as signatures of enhanced collectivity.

Data sources and extraction

Binding-energy quantities used in the thesis analysis were taken from the AME2020 atomic-mass evaluation [6]. Ground-state and low-lying level-structure information was drawn from ENSDF/NNDC and IAEA nuclear-data resources, as documented in the thesis. The extracted variables included Z, N, A, ground-state spin-parity Jπ, total binding energy B, mean binding energy B/A, $E(2_1^+)$, $E(4_1^+)$, $R_{4/2}$, β_2, and selected electromagnetic ground-state quantities.

Analytical procedure

The analysis proceeded in three stages. First, the physical assumptions and domains of applicability of the two models were compared. Second, the numerical quantities reported in the thesis were organized and checked through direct ratios and cross-comparison of the selected nuclei. Third, the model expectations were evaluated against the structural patterns contained in the nuclear-data values. Spreadsheet-based processing was used in the thesis for data organization, ratio calculation, and preparation of comparative tables and graphs. Because the study is a focused review with four nuclei, no inferential statistical test was used; the comparison is descriptive and physics-based.

Scope and limitations

The analysis is restricted to two model frameworks and four nuclei. It does not constitute a global benchmark across the nuclide chart and does not include a full microscopic shell-model or beyond-mean-field calculation. The study also relies on evaluated published data rather than new experimental measurements. These limitations should be considered when generalizing the conclusions.

RESULTS

Structural classification and basic nuclear properties

The selected nuclei separate into two structural groups. ^{152}Sm and ^{168}Er are even–even nuclei outside closed shells and are characterized in the thesis as deformed systems with collective behavior. By contrast, ^{132}Sn has Z = 50 and N = 82, and ^{208}Pb has Z = 82 and N = 126; both are doubly magic and are treated as approximately spherical closed-shell nuclei. All four nuclei have ground-state spin-parity 0^+, consistent with their even–even character.

Table 1. Basic properties and structural classification of the selected nuclei.

Nucleus	Z	N	A	Ground state Jπ	Structural regime
^{152}Sm	62	90	152	0^+	Deformed / transitional
^{168}Er	68	100	168	0^+	Deformed / rotational
^{132}Sn	50	82	132	0^+	Closed shell / nearly spherical
^{208}Pb	82	126	208	0^+	Doubly magic / spherical

Binding-energy systematics

The binding-energy analysis gives total binding energies of 1253.1, 1365.8, 1102.8, and 1636.4 MeV for ^{152}Sm, ^{168}Er, ^{132}Sn, and ^{208}Pb, respectively. Total binding therefore increases substantially with mass number, with ^{208}Pb having the largest total value and ^{132}Sn the smallest. Because total binding scales strongly with the number of nucleons, B/A is more informative for comparing average binding among the four nuclei. The thesis reports B/A values of 8.24, 8.13, 8.36, and 7.87 MeV, respectively; the largest value occurs for ^{132}Sn and the smallest for ^{208}Pb.

Table 2. Binding-energy quantities used in the comparative analysis.

Nucleus	B (MeV)	B/A (MeV)
^{152}Sm	1253.1	8.24
^{168}Er	1365.8	8.13
^{132}Sn	1102.8	8.36
^{208}Pb	1636.4	7.87

Low-lying spectra, rotational indicators, and deformation

The structural indicators display a strong separation between the two nuclear regimes. For ^{152}Sm, $E(2_1^+)$ = 121.8 keV, $E(4_1^+)$ = 366.5 keV, and β_2 = 0.30. For ^{168}Er, the corresponding values are 79.8 keV, 244.7 keV, and β_2 = 0.34. The ratios calculated directly from the tabulated excitation energies are $R_{4/2}$ = 3.01 for ^{152}Sm and 3.07 for ^{168}Er. For ^{132}Sn and ^{208}Pb, the first 2^+ energies are 4041 and 4085 keV, respectively, while β_2 is approximately zero. The corresponding ratios are 1.09 and 1.06. These values establish a pronounced contrast between low-energy collective spectra in the deformed nuclei and high-energy first excitations in the closed-shell nuclei.

Table 3. Low-lying structural and spectroscopic indicators.

Nucleus	β_2	$E(2_1^+)$ (keV)	$E(4_1^+)$ (keV)	$R_{4/2}$	Structural indication
^{152}Sm	0.30	121.8	366.5	3.01	Deformed / collective
^{168}Er	0.34	79.8	244.7	3.07	Rotational / collective
^{132}Sn	≈0	4041	4416	1.09	Closed shell / weak collectivity
^{208}Pb	≈0	4085	4323	1.06	Doubly magic / spherical

Electromagnetic ground-state characteristics

The thesis reports $J\pi = 0^+$ for the ground state of all four nuclei and correspondingly vanishing ground-state magnetic moments. It also emphasizes that a zero spectroscopic quadrupole moment for a 0^+ ground state does not by itself prove that the intrinsic nuclear shape is spherical. Deformation must therefore be interpreted jointly with β_2 and low-lying spectral indicators. In ^{132}Sn and ^{208}Pb, near-zero β_2 is consistent with their closed-shell classification, whereas the nonzero β_2 values of ^{152}Sm and ^{168}Er indicate intrinsic deformation despite the 0^+ ground state.

Model-domain comparison

The thesis-based theoretical comparison assigns complementary strengths to the two frameworks. The liquid-drop model is rated highly for binding energy and nuclear mass systematics and as useful for macroscopic fission behavior, but limited for detailed low-energy levels, rotational bands, and shell-dependent structure. The collective model is rated highly for low-energy levels, rotational bands, deformation, and deformed nuclei, but limited for closed-shell nuclei and global mass properties. The principal result is therefore not that one model universally outperforms the other, but that predictive usefulness depends on both the observable and the structural regime.

Quantitative Cross-Comparison of Model Indicators

To strengthen the comparison, the numerical indicators were considered jointly rather than separately. The thesis data show a consistent separation between the two structural regimes. The deformed nuclei ^{152}Sm and ^{168}Er have low first 2^+ excitation energies (121.8 and 79.8 keV), non-zero quadrupole deformation parameters (0.30 and 0.34), and $R_{4/2}$ values of 3.01 and 3.07,

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respectively. By contrast, ^{132}Sn and ^{208}Pb have much higher first 2^+ energies (4041 and 4085 keV), β_2 values approximately equal to zero, and $R_{4/2}$ values close to unity (1.09 and 1.06). Taken together, these indicators provide mutually consistent evidence for strong collectivity in the first pair and weak collectivity in the second pair.

The binding-energy indicators reveal a complementary pattern. The total binding energies increase from 1102.8 MeV for ^{132}Sn to 1636.4 MeV for ^{208}Pb, largely reflecting the increase in the number of nucleons. Therefore, total binding energy alone is not an adequate measure for comparing relative binding among nuclei of different mass. The binding energy per nucleon gives a more useful comparison: 8.24 MeV for ^{152}Sm, 8.13 MeV for ^{168}Er, 8.36 MeV for ^{132}Sn, and 7.87 MeV for ^{208}Pb. The particularly high B/A of ^{132}Sn and lower B/A of ^{208}Pb illustrate the combined influence of shell closure and increasing Coulomb effects in heavy nuclei.

The combined indicators also clarify why the two models should not be treated as competing universal descriptions. The liquid-drop framework is naturally suited to smooth, global trends in binding and mass-related properties because its principal terms represent volume, surface, Coulomb, asymmetry, and pairing contributions. The collective model, in contrast, becomes especially informative when coherent rotational or vibrational motion produces a characteristic low-energy spectrum. The thesis therefore supports a domain-dependent interpretation: model usefulness follows the physical observable and the structural regime of the nucleus. The complementary domains are summarized in Table 4.

An important methodological distinction is maintained throughout this article. The evaluated nuclear-data values are used as the observational reference set, while the theoretical frameworks are interpreted according to their domains of applicability. No unsupported residual analysis is introduced where the thesis did not report a corresponding model calculation. This preserves the distinction between evaluated experimental information and quantities obtained from model-based calculations.

Table 4. Summary of the complementary domains of the two model frameworks.

Observable / regime	Liquid-drop model	Collective model
Binding-energy and mass trends	Strong domain	Limited
Low-lying excitation spectrum	Limited	Strong domain
Rotational-band behavior	Limited	Strong domain
Quadrupole deformation	Indirect / limited	Strong domain
Closed-shell effects	Incomplete	Limited without microscopic shell treatment
Deformed collective nuclei	Macroscopic background	Strong domain

DISCUSSION

The central finding is a clear separation between macroscopic binding-energy systematics and detailed low-energy structure. The liquid-drop framework captures the broad behavior expected from a saturated nuclear medium: total binding increases with increasing A, while B/A remains within a relatively narrow range. The reported B/A maximum for ^{132}Sn is also consistent with the special stability associated with its closed proton and neutron shells. The lower B/A of ^{208}Pb, despite its much larger total binding energy, illustrates why total B should not be used alone as a measure of relative nuclear stability.

The collective indicators provide a more discriminating structural test. The low $E(2_1^+)$ values and nonzero β_2 values of ^{152}Sm and ^{168}Er identify nuclei capable of sustaining coherent collective motion. Their $R_{4/2}$ values of 3.01 and 3.07 are substantially larger than those of the closed-shell nuclei and are close to the rotational regime. In particular, ^{168}Er has the larger deformation parameter, $\beta_2 = 0.34$, and the lower first 2^+ energy, supporting the thesis interpretation of pronounced rotational collectivity.

In contrast, ^{132}Sn and ^{208}Pb exhibit very high $E(2_1^+)$, $R_{4/2}$ close to unity, and $\beta_2 \approx 0$. These features indicate that low-energy collective motion is strongly suppressed relative to the deformed cases. Their closed-shell character therefore provides information that a purely macroscopic liquid-drop description cannot encode explicitly. This is the regime in which microscopic shell effects become indispensable.

The findings support the study hypothesis in a qualified rather than absolute sense. The liquid-drop model is appropriate for global macroscopic properties, especially binding-energy trends, while the collective model is more informative for deformed nuclei and low-energy rotational structure. However, the evidence does not justify claiming that the liquid-drop model is

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intrinsically superior for closed-shell nuclei in every observable. Rather, the results show that shell closure is an essential structural ingredient that neither of the two selected frameworks fully represents on its own.

An important methodological point concerns the relation between the reported calculations and evaluated data. The thesis identifies AME2020 as the source for the binding-energy values and ENSDF/NNDC/IAEA as sources for structural data. Accordingly, this article treats these quantities as the evaluated dataset used to test and interpret the model frameworks, rather than introducing model-to-data residuals that are not explicitly reported in the thesis. This preserves the distinction between calculated model quantities and externally evaluated nuclear information.

The study has a useful master's-level methodological contribution: it demonstrates that model selection should follow the physics of the observable. A macroscopic model can reproduce broad global trends while missing shell-sensitive details; conversely, a collective description can explain rotational spectra without providing a complete account of nuclear masses. The complementary use of models is therefore a consequence of the multifaceted nature of nuclear structure.

The main limitations are the small sample of four nuclei, the restriction to two model frameworks, and the absence of new experimental measurements or a full microscopic benchmark. Future work can extend the analysis to transitional nuclei, nuclei farther from stability, electromagnetic transition probabilities such as B(E2), and explicit shell-model or macroscopic–microscopic calculations.

Overall, the thesis evidence supports a complementary rather than hierarchical interpretation of the two models. The selected nuclei deliberately span deformed/collective and closed-shell regimes, allowing the same set of observables to demonstrate where macroscopic and collective descriptions succeed or become incomplete. This cross-regime comparison is the principal analytical value of the study.

CONCLUSION

This comparative analysis of ^{152}Sm, ^{168}Er, ^{132}Sn, and ^{208}Pb demonstrates that the usefulness of a nuclear model is strongly dependent on the observable and the structural regime. The liquid-drop framework provides a coherent macroscopic description of global binding-energy behavior, including the observed increase of total binding with mass number and the relatively narrow range of B/A. The collective model is more effective for interpreting the low-energy structure of deformed nuclei, where nonzero β_2, low $E(2_1^+)$, and relatively large $R_{4/2}$ values reveal pronounced rotational collectivity. Conversely, the high first-excitation energies, near-zero deformation, and small $R_{4/2}$ values of ^{132}Sn and ^{208}Pb identify strong closed-shell character that requires explicit microscopic treatment. The research objectives are therefore met through a complementary interpretation: neither model alone is sufficient for all aspects of nuclear structure, and a more complete description requires the combined use of macroscopic, collective, and shell-sensitive approaches.

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