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Kafi-Pawat Family of Distributions with Applications to COVID-19, Labor Economics, Medical, and Environmental Data

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ABSTRACT: This article presents a novel and flexible family of continuous probability distributions, namely the Kafi-Pawat family of distributions. The Kafi-Pawat family is characterized by two parameters, playing an important role in controlling the shape of the hazard rate function, thereby enhancing its flexibility for modeling diverse data behaviors. We derive key distributional functions of the Kafi-Pawat family, including its hazard rate function. To demonstrate the flexibility and practical utility of the proposed family, we introduce and study several members of the Kafi-Pawat family. The hazard rate functions of all distributions within the Kafi-Pawat family can be monotone or non-monotone, highlighting their flexibility. Parameter estimation is conducted via the method of maximum likelihood. Since the maximum likelihood estimators cannot be obtained in closed form, we employ numerical optimization techniques to obtain the fitted parameter values. The final section is to apply the established distributions to the real-world datasets. Comparative analyses among the considered distributions are performed to exhibit their potential as flexible and effective tool for modeling uncertainty.

KEYWORDS: Average estimate, Exponential distribution, Heavy-tailed, Inverse Burr, Maximum likelihood estimation, Quantile function

INTRODUCTION

In statistical theory, developing new probability distributions is a well-established and constantly evolving field. Developing and applying probability distributions to model uncertain events remains a dynamic and an expanding area of research. A common approach in this area involves extending existing distributions by introducing additional parameters, thereby enhancing the model's flexibility to capture a broader range of real-world phenomena. In continuous probability distributions, flexibility usually refers to the ability of a model to accommodate various shapes of the hazard rate function, which is crucial in survival analysis, reliability engineering, and related fields.

One of the simplest and most widely used non-negative continuous distributions is exponential distribution, defined by a single parameter. Despite its mathematical simplicity and interpretability, the exponential distribution assumes a constant hazard rate, limiting its applicability to datasets characterized by increasing, decreasing, or non-monotonic hazard behaviors [1]. To address this limitation, more flexible models were introduced, such as Gama and Weibull distributions. These distributions generalize the exponential model and are able to capture increasing and decreasing hazard rate functions [2]. They also have heavier tails compared to the exponential distribution, allowing for better modeling of extreme events. However, distributions like Gamma, Weibull, and Pareto still have limitations. In particular, they are only able to have monotonic hazard rate functions, and thereby unable to capture data with non-monotonic (e.g., unimodal) hazard behaviors [2-3].

In this study, we introduce a new family of continuous distributions called the Kafi-Pawat (KP) family of distributions. While several distribution families have been proposed previously, such as [4-12], not all their members exhibit sufficient flexibility. The primary objective of this article is to present a new class of continuous distributions for positive random variables that offers greater flexibility than widely used models such as the gamma, Pareto, and Weibull distributions. All continuous distributions belong to the KP family are capable of generating data with both monotonic and non-monotonic hazard rate functions. This level of flexibility is not typically found in classic distributions as well as distributions in other families. This feature sets the distributions in KP family apart from widely used models, including the exponential [1], Weibull [2], Pareto [3], Lindley [13], Gompertz [14], Bilal [15], Rayleigh [16], and Muth [17] distributions, all of which are limited to monotonic hazard rate behaviors.

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KAFI-PAWAT FAMILY OF DISTRIBUTIONS

A. Formulation of the Kafi-Pawat Family

The conception of the Kafi-Pawat (KP) family of distributions was initially motivated by an investigation of the following rational function defined on the positive real axis:

$$g_1(x) = \frac{x}{1+x}, \qquad x \geq 0, \tag{1}$$

in which plot is given in Figure 1.

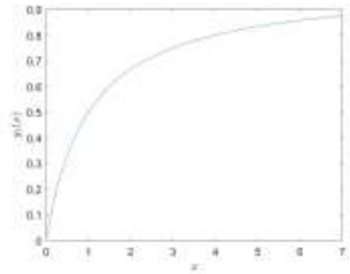

Figure 1. The plot of function g_1 on the interval [0, 7].

It is evident that the function $g_1(x)$ exhibits the characteristic shape of a cumulative distribution function (CDF) and, as such, may serve as a potential CDF for a positive real-valued random variable. However, since $g_1(x)$ does not contain any parameters, it lacks the flexibility required to model a wide range of real datasets. Moreover, since KP is a family of distributions, it is necessary to generalize the Equation (1) to get the general form of CDF for a family of distributions.

In this context, the function $g_1(x)$ is modified to derive the CDF of the proposed family of continuous distributions, referred to as the Kafi-Pawat (KP) family of distributions, which is expected to produce flexible distributions. The CDF of the KP family is constructed as follows:

1. Consider a function defined in Equation (1). By adding parameter $\beta > 0$, Equation (1) becomes
$$g_2(x) = \frac{\frac{x}{\beta}}{1+\frac{x}{\beta}}, \quad x \geq 0;\ \beta > 0. \tag{2}$$
The value of β must be positive in order to preserve the CDF curve as already formed by the function $g_1(x)$.
2. By powering a positive real number α to the term $\frac{x}{\beta}$, Equation (2) transforms into Equation (3).
$$g_3(x) = \frac{\left(\frac{x}{\beta}\right)^\alpha}{1+\left(\frac{x}{\beta}\right)^\alpha}, \quad x \geq 0; \alpha > 0; \beta > 0. \tag{3}$$
The value of α must also be positive in order to preserve the CDF curve as already formed by the function $g_2(x)$.
3. By replacing the term $\frac{x}{\beta}$ with a function $G(x) = \frac{f(x)}{\beta}$, Equation (3) transforms into Equation (4).
$$g_4(x) = \frac{\left(\frac{f(x)}{\beta}\right)^\alpha}{1+\left(\frac{f(x)}{\beta}\right)^\alpha}, \quad x > 0; \alpha > 0; \beta > 0. \tag{4}$$

However, the conditions on $f(x)$ must be explicitly specified to ensure that Equation (4) satisfies the properties of a CDF. The formal definition of the KP family, together with the characterization of $f(x)$, is presented in Definition 1.

Definition 1. A continuous probability distribution for a positive random variable X is said to belong to the Kafi-Pawat family of distributions if its cumulative distribution function (CDF) can be expressed as follows:

$$F_{\text{KP}}(x) = \begin{cases} 0, & x \leq 0 \\ \frac{\left(\frac{f(x)}{\beta}\right)^\alpha}{1+\left(\frac{f(x)}{\beta}\right)^\alpha}, & x > 0 \end{cases}, \tag{5}$$

where $\alpha > 0$, $\beta > 0$, and $f(x)$ is a positive, non-decreasing, invertible, differentiable, and continuous function over $x > 0$, such that $\lim_{x \to 0^+} f(x) = 0$ and $\lim_{x\to\infty} f(x) = \infty$.

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Proposition 1. The function presented in Equation (5) satisfies all the essential properties of a CDF, that is,

1. $0 \le F_{\mathrm{KP}}(x) \le 1$, for all $x \in \mathbb{R}$.
2. $F_{\mathrm{KP}}(x)$ is a non-decreasing function over $\mathbb{R}$.
3. $F_{\mathrm{KP}}(x)$ is a right-continuous function for all $x \in \mathbb{R}$.
4. $\lim_{x\to\infty} F_{\mathrm{KP}}(x) = 1$ and $\lim_{x\to-\infty} F_{\mathrm{KP}}(x) = 0$.

Proof. First, it will be shown that $0 \le F_{\mathrm{KP}}(x) \le 1$, for all $x \in \mathbb{R}$. (i) For $x \le 0$, $F_{\mathrm{KP}}(x) = 0$ and thereby belongs to $[0,1]$. (ii) Consider $F_{\mathrm{KP}}(x)$ for $x > 0$. For any $\alpha > 0$ and $\beta > 0$, the following equivalences hold.

$$1 + \left(\frac{f(x)}{\beta}\right)^{\alpha} > 1 \Leftrightarrow 0 < \frac{1}{1 + \left(\frac{f(x)}{\beta}\right)^{\alpha}} < 1 \Leftrightarrow 0 < \frac{\left(\frac{f(x)}{\beta}\right)^{\alpha}}{1 + \left(\frac{f(x)}{\beta}\right)^{\alpha}} < 1$$

The above expression implies that $F_{\mathrm{KP}}(x) \in (0,1)$ for all $x > 0$. However, the interval $(0,1) \subseteq [0,1]$, and thus $F_{\mathrm{KP}}(x)$ also belongs to the closed interval $[0,1]$. From case (i) and (ii), we conclude that $F_{\mathrm{KP}}(x) \in [0,1]$ for all $x \in \mathbb{R}$.

Next, it will be shown that $F_{\mathrm{KP}}(x)$ is a non-decreasing function over $\mathbb{R}$. The function $F_{\mathrm{KP}}(x)$ is non-decreasing if $F'_{\mathrm{KP}}(x) \ge 0$. Differentiate $F_{\mathrm{KP}}(x)$ in Equation (5) with respect to x, we obtain:

$$F'_{\mathrm{KP}}(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{\alpha\left(f(x)\right)^{\alpha-1} f'(x)\beta^{\alpha}}{\left[\left(f(x)\right)^{\alpha} + \beta^{\alpha}\right]^2}, & x > 0 \end{cases}.$$

Since α and β are positive, and $f(x)$ is non-decreasing function, the expression $\frac{\alpha(f(x))^{\alpha-1} f'(x)\beta^{\alpha}}{\left[(f(x))^{\alpha} + \beta^{\alpha}\right]^2}$ is positive. Hence, the function $F'_{\mathrm{KP}}(x)$ is non-negative and this implies $F_{\mathrm{KP}}(x)$ is non-decreasing.

Third, it will be shown that $F_{\mathrm{KP}}(x)$ is a right-continuous function for all $x \in \mathbb{R}$. Consider the limit from the right of $F_{\mathrm{KP}}(x)$ as x tends to a given real number c.

- For $c < 0$, yields $\lim_{x\to c^+} F_{\mathrm{KP}}(x) = 0 = F_{\mathrm{KP}}(c)$.
- For $c = 0$, yields

$$\lim_{x\to 0^+} F_{\mathrm{KP}}(x) = \lim_{x\to 0^+} \left(\frac{\left(\frac{f(x)}{\beta}\right)^{\alpha}}{1 + \left(\frac{f(x)}{\beta}\right)^{\alpha}} \right) = \frac{\left(\frac{\lim_{x\to 0^+} f(x)}{\beta}\right)^{\alpha}}{1 + \left(\frac{\lim_{x\to 0^+} f(x)}{\beta}\right)^{\alpha}} = \frac{0^{\alpha}}{1 + 0^{\alpha}} = 0 = F_{\mathrm{KP}}(0).$$

- For $c > 0$, yields

$$\lim_{x\to c^+} F_{\mathrm{KP}}(x) = \lim_{x\to c^+} \left(\frac{\left(\frac{f(x)}{\beta}\right)^{\alpha}}{1 + \left(\frac{f(x)}{\beta}\right)^{\alpha}} \right) = \frac{\left(\frac{f(c)}{\beta}\right)^{\alpha}}{1 + \left(\frac{f(c)}{\beta}\right)^{\alpha}} = F_{\mathrm{KP}}(c).$$

Since $\lim_{x\to c^+} F_{\mathrm{KP}}(x) = F_{\mathrm{KP}}(c)$ for all $c \in \mathbb{R}$, then $F_{\mathrm{KP}}(x)$ is a right-continuous function.

Fourth, it will be shown that $\lim_{x\to\infty} F_{\mathrm{KP}}(x) = 1$ and $\lim_{x\to-\infty} F_{\mathrm{KP}}(x) = 0$. We are given $\lim_{x\to\infty} f(x) = \infty$, and thereby $\lim_{x\to\infty} \frac{f(x)}{\beta} = \infty$. Hence,

$$\lim_{x\to\infty} F_{\mathrm{KP}}(x) = \lim_{x\to\infty} \left(\frac{\left(\frac{f(x)}{\beta}\right)^{\alpha}}{1+\left(\frac{f(x)}{\beta}\right)^{\alpha}} \right) = \lim_{x\to\infty} \left(\frac{1}{\left(\frac{f(x)}{\beta}\right)^{-\alpha} + 1} \right) = \frac{1}{0+1} = 1,$$

$$\lim_{x\to-\infty} F_{\mathrm{KP}}(x) = \lim_{x\to-\infty} 0 = 0.$$

As the function presented in Equation (5) satisfies the four defining properties of a CDF, it can be concluded that $F_{\mathrm{KP}}(x)$ constitutes the CDF of a random variable X. ■

Given the CDF of the Kafi-Pawat (KP) family as defined in Equation (5), the corresponding probability density function (PDF) can be derived as follows:

$$f_{\text{KP}}(x) = \frac{d}{dx}[F_{\text{KP}}(x)] = \frac{d}{dx}\left[\frac{\left(\frac{f(x)}{\beta}\right)^{\alpha}}{1+\left(\frac{f(x)}{\beta}\right)^{\alpha}}\right] = \frac{\alpha(f(x))^{\alpha-1}f'(x)\beta^{\alpha}}{\left[(f(x))^{\alpha}+\beta^{\alpha}\right]^{2}}, \quad x > 0. \tag{6}$$

The survival function (SF) and hazard rate function (HRF) of KP family are given, respectively, by Equations (7) and (8).

$$S_{\text{KP}}(x) = 1 - F_{\text{KP}}(x) = 1 - \frac{\left(\frac{f(x)}{\beta}\right)^{\alpha}}{1+\left(\frac{f(x)}{\beta}\right)^{\alpha}} = \frac{1}{1+\left(\frac{f(x)}{\beta}\right)^{\alpha}}, \quad x > 0. \tag{7}$$

$$h_{\text{KP}}(x) = \frac{f_{\text{KP}}(x)}{S_{\text{KP}}(x)} = \frac{\frac{\alpha(f(x))^{\alpha-1}f'(x)\beta^{\alpha}}{\left[(f(x))^{\alpha}+\beta^{\alpha}\right]^{2}}}{\frac{1}{1+\left(\frac{f(x)}{\beta}\right)^{\alpha}}} = \frac{\frac{\alpha(f(x))^{\alpha-1}f'(x)\beta^{\alpha}}{\left[(f(x))^{\alpha}+\beta^{\alpha}\right]^{2}}}{\frac{1}{\frac{(f(x))^{\alpha}+\beta^{\alpha}}{\beta^{\alpha}}}} = \frac{\alpha(f(x))^{\alpha-1}f'(x)}{(f(x))^{\alpha}+\beta^{\alpha}}, \quad x > 0. \tag{8}$$

B. Some Members of Kafi-Pawat Family and Their Flexibility

This subsection introduces several members of Kafi-Pawat (KP) family of distributions alongside their corresponding distributional functions. Specifically, four probability distributions are presented, three of which are new distributions.

1) Inverse Burr (IB) distribution [18], if $f(x) = x$ for $x > 0$.

CDF: $F_{\text{IB}}(x) = \begin{cases} 0, & x \le 0 \\ \frac{\left(\frac{x}{\beta}\right)^{\alpha}}{1+\left(\frac{x}{\beta}\right)^{\alpha}}, & x > 0 \end{cases}$; PDF: $f_{\text{IB}}(x) = \frac{\alpha\beta^{\alpha}x^{\alpha-1}}{[\beta^{\alpha}+x^{\alpha}]^{2}}$, $x > 0$.

SF: $S_{\text{IB}}(x) = \frac{1}{1+\left(\frac{x}{\beta}\right)^{\alpha}}$, $x > 0$; HRF: $h_{\text{IB}}(x) = \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}+x^{\alpha}}$, $x > 0$.

2) Kafi-Pawat shifted exponential (KPSE) distribution (New), if $f(x) = e^{x} - 1$ for $x > 0$.

CDF: $F_{\text{KPSE}}(x) = \begin{cases} 0, & x \le 0 \\ \frac{\left(\frac{e^{x}-1}{\beta}\right)^{\alpha}}{1+\left(\frac{e^{x}-1}{\beta}\right)^{\alpha}}, & x > 0 \end{cases}$; PDF: $f_{\text{KPSE}}(x) = \frac{\alpha e^{x}\left(\frac{e^{x}-1}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{e^{x}-1}{\beta}\right)^{\alpha}\right]^{2}}$, $x > 0$.

SF: $S_{\text{KPSE}}(x) = \frac{1}{1+\left(\frac{e^{x}-1}{\beta}\right)^{\alpha}}$, $x > 0$; HRF: $h_{\text{KPSE}}(x) = \frac{\alpha e^{x}\left(\frac{e^{x}-1}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{e^{x}-1}{\beta}\right)^{\alpha}\right]}$, $x > 0$.

3) Kafi-Pawat shifted hyperbolic cosine (KPSHC) distribution (New), if $f(x) = \cosh(x) - 1$ for $x > 0$.

CDF: $F_{\text{KPSHC}}(x) = \begin{cases} 0, & x \le 0 \\ \frac{\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha}}{1+\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha}}, & x > 0 \end{cases}$; PDF: $f_{\text{KPSHC}}(x) = \frac{\alpha\sinh(x)\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha}\right]^{2}}$, $x > 0$.

SF: $S_{\text{KPSHC}}(x) = \frac{1}{1+\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha}}$, $x > 0$; HRF: $h_{\text{KPSHC}}(x) = \frac{\alpha\sinh(x)\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha}\right]}$, $x > 0$.

4) Kafi-Pawat logarithmic (KPL) distribution (New), if $f(x) = \ln(x + 1)$ for $x > 0$.

CDF: $F_{\text{KPL}}(x) = \begin{cases} 0, & x \le 0 \\ \frac{\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha}}{1+\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha}}, & x > 0 \end{cases}$; PDF: $f_{\text{KPL}}(x) = \frac{\frac{\alpha}{x+1}\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha}\right]^{2}}$, $x > 0$.

SF: $S_{\text{KPL}}(x) = \frac{1}{1+\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha}}$, $x > 0$; HRF: $h_{\text{KPL}}(x) = \frac{\frac{\alpha}{x+1}\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha}\right]}$, $x > 0$.

FLEXIBILITY OF KAFI-PAWAT FAMILY

As stated in Section 1, the primary objective of this study is to introduce a family of flexible continuous distributions. To this end, an analysis of the hazard rate functions (HRFs) of the distributions within the KP family is undertaken. If the HRFs of all members exhibit both monotonic and non-monotonic behaviors, the KP family is deemed flexible. Section 2 outlines several members of the

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KP family, namely the Inverse Burr, Kafi-Pawat Shifted Exponential, Kafi-Pawat Shifted Hyperbolic Cosine, and Kafi-Pawat Logarithmic distributions. The corresponding hazard rate analyses are presented in separate subsections below.
It is important to note that there are two methods that can be considered to demonstrate whether the HRF of a given distribution is monotone or non-monotone. The choice of method depends on the mathematical expression of the HRF itself. If the HRF has a closed-form expression and is sufficiently tractable, monotonicity can be established analytically using relevant definitions or theorems. Otherwise, a graphical approach is employed, whereby several parameter combinations are selected and the corresponding HRFs are plotted.

C. Flexibility of Inverse Burr Distribution

According to Section 2, the hazard rate function (HRF) of inverse Burr distribution is given as follows:

$$h_{\mathrm{IB}}(x) = \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}+x^{\alpha}},\ x > 0. \tag{9}$$

The HRF of the inverse Burr distribution, as defined in Equation (9), exhibits various forms depending on the values of its parameters. In this distribution, β and α are scale and shape parameters, respectively [18]. In particular, the shape parameter α plays a crucial role in determining the overall behavior of the HRF. To illustrate these different behaviors, the analysis of the inverse Burr distribution's hazard rate function is presented on a case-by-case basis, highlighting the influence of the shape parameter on the distribution's failure rate characteristics.
Case 1: $0 < \alpha \leq 1$ and $\beta > 0$. In this case, the HRF given in Equation (9) is a decreasing function. The proof is given as follows:
The first derivative of HRF in Equation (9) is

$$h'_{\mathrm{IB}}(x) = \frac{\alpha(\alpha-1)x^{\alpha-2}(\beta^{\alpha}+x^{\alpha}) - \alpha^2 x^{2\alpha-2}}{(\beta^{\alpha}+x^{\alpha})^2}, \qquad x > 0.$$

It is obvious that denominator $(\beta^{\alpha}+x^{\alpha})^2$ is strictly positive. Meanwhile, since $0 < \alpha \leq 1$, $\beta > 0$, and $x > 0$, then $\beta^{\alpha}+x^{\alpha} > 0$ and $-1 < \alpha - 1 \leq 0$. This implies
$\alpha(\alpha-1)x^{\alpha-2}(\beta^{\alpha}+x^{\alpha}) \leq 0$ and $-\alpha^2 x^{2\alpha-2} < 0$,
and thereby

$$\alpha(\alpha-1)x^{\alpha-2}(\beta^{\alpha}+x^{\alpha}) - \alpha^2 x^{2\alpha-2} < 0.$$

Therefore,

$$h'_{\mathrm{IB}}(x) = \frac{\alpha(\alpha-1)x^{\alpha-2}(\beta^{\alpha}+x^{\alpha}) - \alpha^2 x^{2\alpha-2}}{(\beta^{\alpha}+x^{\alpha})^2} < 0.$$

This concludes that for $0 < \alpha \leq 1$, the HRF of the inverse Burr distribution is a decreasing function. ■
The following is the asymptotic behavior of its hazard rate function.

$$h_{\mathrm{IB}}(x;\alpha,\beta) = \begin{cases} \infty, & \text{if } x \to 0 \\ \dfrac{1}{\beta}, & \text{if } x \to 0 \text{ and } \alpha = 1 \\ 0, & \text{if } x \to \infty \end{cases}$$

In particular if $\alpha = 1$, the HRF of $\mathrm{IB}(\alpha,\beta)$ is bounded below and above by zero and $\frac{1}{\beta}$, respectively.
Case 2: $\alpha > 1$ and $\beta > 0$. In this case, the HRF given in Equation (9) is a unimodal function. The proof is given as follows:
First, the stationary point of function $h_{\mathrm{IB}}(x)$ must be obtained. The stationary point x_s of $h_{\mathrm{IB}}(x)$ is the solution of the following equation.

$$h'_{\mathrm{IB}}(x_s) = 0, \qquad x > 0.$$

$$\Leftrightarrow \frac{\alpha(\alpha-1){x_s}^{\alpha-2}(\beta^{\alpha}+{x_s}^{\alpha}) - \alpha^2 {x_s}^{2\alpha-2}}{(\beta^{\alpha}+{x_s}^{\alpha})^2} = 0$$

$$\Leftrightarrow (\alpha-1)(\beta^{\alpha}+{x_s}^{\alpha}) - \alpha {x_s}^{\alpha} = 0$$

$$\Leftrightarrow (\alpha-1)\beta^{\alpha} - {x_s}^{\alpha} = 0 \Leftrightarrow x_s = [(\alpha-1)\beta^{\alpha}]^{\frac{1}{\alpha}} = \beta(\alpha-1)^{\frac{1}{\alpha}}$$

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Hence, the stationary point of $h_{\mathrm{IB}}(x)$ is $[(\alpha-1)\beta^{\alpha}]^{\frac{1}{\alpha}}$ and does maximize $h_{\mathrm{IB}}(x)$ on $x>0$. The next step is to show that the function $h_{\mathrm{IB}}(x)$ is increasing on the interval $\left(0,\ \beta(\alpha-1)^{\frac{1}{\alpha}}\right)$ and is decreasing on the interval $\left(\beta(\alpha-1)^{\frac{1}{\alpha}},\ \infty\right)$.

Having $x\in\left(0,\ \beta(\alpha-1)^{\frac{1}{\alpha}}\right)$, it implies

$$\begin{gathered}x<[(\alpha-1)\beta^{\alpha}]^{\frac{1}{\alpha}} \Leftrightarrow x^{\alpha}<(\alpha-1)\beta^{\alpha}\\ \Leftrightarrow x^{\alpha}+(\alpha-1)x^{\alpha}<(\alpha-1)\beta^{\alpha}+(\alpha-1)x^{\alpha}\\ \Leftrightarrow \alpha x^{\alpha}<(\alpha-1)(\beta^{\alpha}+x^{\alpha})\\ \Leftrightarrow \alpha^{2}x^{2\alpha-2}<\alpha(\alpha-1)(\beta^{\alpha}+x^{\alpha})x^{\alpha-2}\\ \Leftrightarrow \alpha(\alpha-1)(\beta^{\alpha}+x^{\alpha})x^{\alpha-2}-\alpha^{2}x^{2\alpha-2}>0\\ \Leftrightarrow \frac{\alpha(\alpha-1)x^{\alpha-2}(\beta^{\alpha}+x^{\alpha})-\alpha^{2}x^{2\alpha-2}}{(\beta^{\alpha}+x^{\alpha})^{2}}>0 \Leftrightarrow h'_{\mathrm{IB}}(x)>0\end{gathered}$$

For $x\in\left(0,\ \beta(\alpha-1)^{\frac{1}{\alpha}}\right)$, the first derivative of $h_{\mathrm{IB}}(x)$ is positive. This indicates that $h_{\mathrm{IB}}(x)$ is an increasing function on $\left(0,\ \beta(\alpha-1)^{\frac{1}{\alpha}}\right)$.

Next, having $x\in\left(\beta(\alpha-1)^{\frac{1}{\alpha}},\ \infty\right)$, it implies

$$\begin{gathered}x>[(\alpha-1)\beta^{\alpha}]^{\frac{1}{\alpha}} \Leftrightarrow x^{\alpha}>(\alpha-1)\beta^{\alpha}\\ \Leftrightarrow x^{\alpha}+(\alpha-1)x^{\alpha}>(\alpha-1)\beta^{\alpha}+(\alpha-1)x^{\alpha}\\ \Leftrightarrow \alpha x^{\alpha}>(\alpha-1)(\beta^{\alpha}+x^{\alpha})\\ \Leftrightarrow \alpha^{2}x^{2\alpha-2}>\alpha(\alpha-1)(\beta^{\alpha}+x^{\alpha})x^{\alpha-2}\\ \Leftrightarrow \alpha(\alpha-1)(\beta^{\alpha}+x^{\alpha})x^{\alpha-2}-\alpha^{2}x^{2\alpha-2}<0\\ \Leftrightarrow \frac{\alpha(\alpha-1)x^{\alpha-2}(\beta^{\alpha}+x^{\alpha})-\alpha^{2}x^{2\alpha-2}}{(\beta^{\alpha}+x^{\alpha})^{2}}<0 \Leftrightarrow h'_{\mathrm{IB}}(x)<0\end{gathered}$$

For $x\in\left(\beta(\alpha-1)^{\frac{1}{\alpha}},\ \infty\right)$, the first derivative of $h_{\mathrm{IB}}(x)$ is negative. This indicates that $h_{\mathrm{IB}}(x)$ is a decreasing function on $\left(\beta(\alpha-1)^{\frac{1}{\alpha}},\ \infty\right)$. Thus, since there is a point $x_s=\beta(\alpha-1)^{\frac{1}{\alpha}}$ such that $h_{\mathrm{IB}}(x)$ increases on $(0,x_s)$, $h'_{\mathrm{IB}}(x_s)=0$, and $h_{\mathrm{IB}}(x)$ decreases on (x_s,∞), the HRF of the inverse Burr distribution is unimodal (upside-down bathtub). ■

Therefore, based on the results from all considered cases, the inverse Burr distribution exhibits potential for effectively modeling and generating data with monotone (decreasing) and non-monotone (unimodal) hazard rate characteristics.

D. Flexibility of Kafi-Pawat Shifted Exponential Distribution

According to Section 2, the hazard rate function (HRF) of Kafi-Pawat shifted exponential (KPSE) distribution is given as follows:

$$h_{\mathrm{KPSE}}(x)=\frac{\alpha e^{x}\left(\frac{e^{x}-1}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{e^{x}-1}{\beta}\right)^{\alpha}\right]},\ x>0. \tag{10}$$

The HRF of the KPSE distribution, as defined in Equation (10), exhibits various forms depending on the values of its parameters. In this distribution, both α and β are shape parameters. The analysis of the KPSE hazard rate function is presented on a case-by-case basis as follows:

Case 1: $\alpha=1$ and $\beta=1$. In this case, Equation (10) simplifies to

$$h_{\mathrm{KPSE}}(x)=\frac{e^{x}\left(\frac{e^{x}-1}{1}\right)^{1-1}}{\left[1+\left(\frac{e^{x}-1}{1}\right)^{1}\right]}=\frac{e^{x}}{e^{x}}=1,\quad x>0.$$

Hence, in this case, the hazard rate function of KPSE distribution is only a constant function.

Case 2: $\alpha=1$ and $\beta>0$. In this case, Equation (10) simplifies to

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$$h_{\mathrm{KPSE}}(x;\beta)=\frac{e^{x}\left(\frac{e^{x}-1}{\beta}\right)^{1-1}}{\beta\left[1+\frac{e^{x}-1}{\beta}\right]}=\frac{e^{x}}{e^{x}+\beta-1}. \quad (11)$$

If $0<\beta\leq 1$, the HRF given in Equation (11) is a monotonically decreasing function. The proof is given as follows:
The first derivative of HRF in Equation (11) is

$$h'_{\mathrm{KPSE}}(x;\beta)=\frac{e^{x}(\beta-1)}{(e^{x}+\beta-1)^{2}}, \qquad x>0.$$

It is obvious that denominator $(e^{x}+\beta-1)^{2}$ is strictly positive. Meanwhile, since $0<\beta\leq 1$, then $-1<\beta-1\leq 0$. This implies the numerator $e^{x}(\beta-1)$ is non-positive. Therefore,

$$h'_{\mathrm{KPSE}}(x;\beta)=\frac{e^{x}(\beta-1)}{(e^{x}+\beta-1)^{2}}\leq 0.$$

This concludes that for $0<\beta\leq 1$, the HRF of the KPSE distribution is a monotonically decreasing function. ■
If $\beta>1$, the HRF given in Equation (11) is an increasing function. The proof is given as follows:
The first derivative of HRF in Equation (11) is

$$h'_{\mathrm{KPSE}}(x;\beta)=\frac{e^{x}(\beta-1)}{(e^{x}+\beta-1)^{2}}, \qquad x>0.$$

It is obvious that denominator $(e^{x}+\beta-1)^{2}$ is strictly positive. Meanwhile, since $\beta>1$, then $\beta-1>0$. This implies the numerator $e^{x}(\beta-1)$ is strictly positive as well. Therefore,

$$h'_{\mathrm{KPSE}}(x;\beta)=\frac{e^{x}(\beta-1)}{(e^{x}+\beta-1)^{2}}>0.$$

This concludes that for $\beta>1$, the HRF of the KPSE distribution is an increasing function. ■
Here is the asymptotic behavior of the HRF of $\mathrm{KPSE}(1,\beta)$.

$$h_{\mathrm{KPSE}}(x;\beta)=\begin{cases}1, & \text{if } x\to\infty\\ \frac{1}{\beta}, & \text{if } x\to 0\end{cases}.$$

For $0<\beta\leq 1$, the HRF is a monotonically decreasing function, and it is bounded below and above, respectively, by one and $\frac{1}{\beta}$.

For $\beta>1$, the HRF is an increasing function, and it is bounded below and above, respectively, by $\frac{1}{\beta}$ and one.

Case 3: $\alpha>0$ and $\beta=1$. In this case, Equation (10) simplifies to

$$h_{\mathrm{KPSE}}(x;\alpha)=\frac{\alpha e^{x}(e^{x}-1)^{\alpha-1}}{[1+(e^{x}-1)^{\alpha}]}. \quad (12)$$

The HRF in Equation (12) can be bathtub and unimodal shapes. However, since Equation (12) is too complex, a graphical approach becomes relevant to be used to show the curves produced by this HRF. Figures 2 and 3 present the plot of the hazard rate function (HRF) for $\mathrm{KPE}(\alpha,1)$.

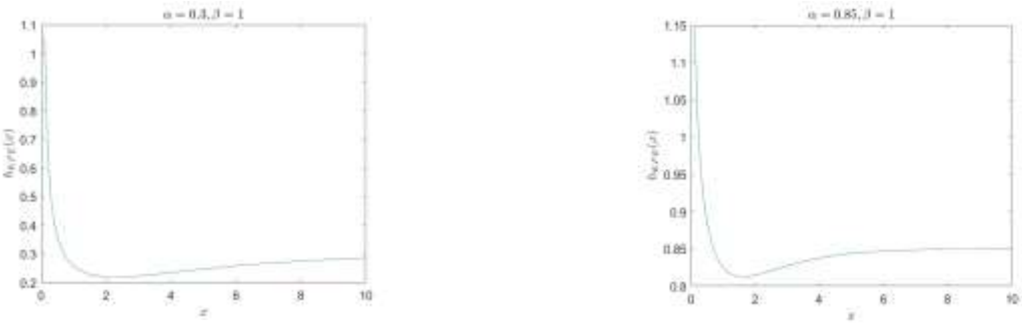

Figure 2. The hazard rate of $\mathrm{KPSE}(\alpha,1)$ for $0<\alpha\leq 1$.

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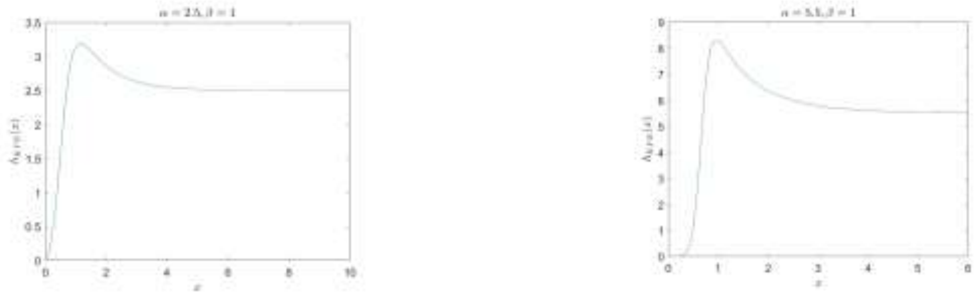

Figure 3. The hazard rate of KPSE$(\alpha, 1)$ for $\alpha > 1$.

Here is the asymptotic behavior of its HRF

$$h_{\text{KPSE}}(x;\alpha) = \begin{cases} \alpha, & \text{if } x \to \infty \\ \infty, & \text{if } x \to 0 \text{ and } \alpha \leq 1 \\ 0, & \text{if } x \to 0 \text{ and } \alpha > 1 \end{cases}.$$

For $0 < \alpha \leq 1$, the HRF has a bathtub shape, and for $\alpha > 1$, the HRF has a unimodal shape.

Case 4: $\alpha > 0$, $\beta > 0$, and $\alpha, \beta \neq 1$. In this case, Equation (10) remains the same and the plots are given in Figures 4 and 5. The hazard rate of KPSE(α, β) can be increasing, monotonically decreasing, bathtub, and unimodal.

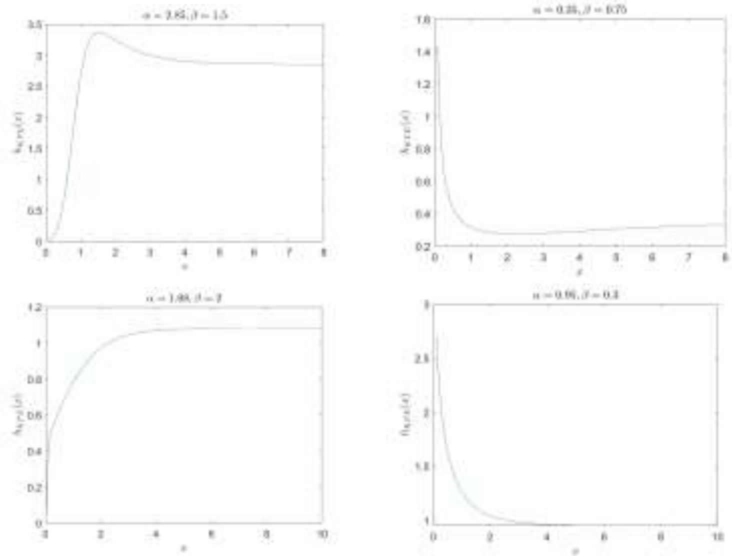

Figure 4. The hazard rate of KPSE(α, β) for $\alpha, \beta > 1$ and $\alpha, \beta < 1$.

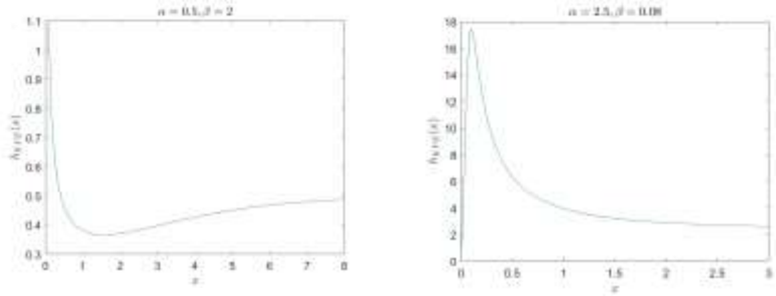

Figure 5. The hazard rate of KPSE(α, β) for $\alpha < 1$ and $\beta > 1$, and $\alpha > 1$ and $\beta < 1$.

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Therefore, based on the results from all considered cases, the KPSE distribution exhibits potential for effectively modeling and generating data with monotone (constant, increasing, monotonically decreasing) and non-monotone (bathtub and unimodal) hazard rates.

E. Flexibility of Kafi-Pawat Shifted Hyperbolic Cosine Distributions

According to Section 2, the hazard rate function (HRF) of Kafi-Pawat shifted hyperbolic cosine (KPSHC) distribution is given as follows:

$$h_{\text{KPSHC}}(x) = \frac{\alpha \sinh(x)\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{\cosh(x)-1}{\beta}\right)^{\alpha}\right]},\ x > 0. \tag{13}$$

The HRF of the KPSHC distribution, as defined in Equation (13), exhibits various shapes depending on the values of its parameters. In this distribution, both α and β are shape parameters. The plots of KPSHC hazard rate function are given in Figures 6 and 7. The hazard rate of KPSHC(α, β) can be monotonically increasing, bathtub, and unimodal.

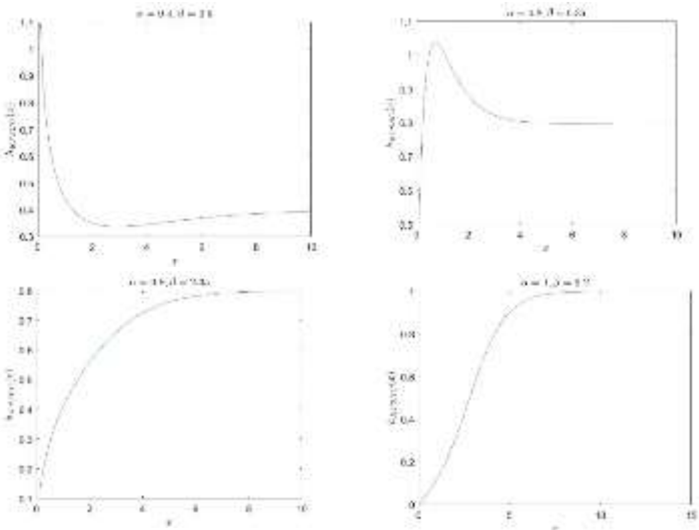

Figure 6. The plots of hazard rate function of KPSHC(α, β) for $0 < \alpha \leq 1$ and $\beta > 0$.

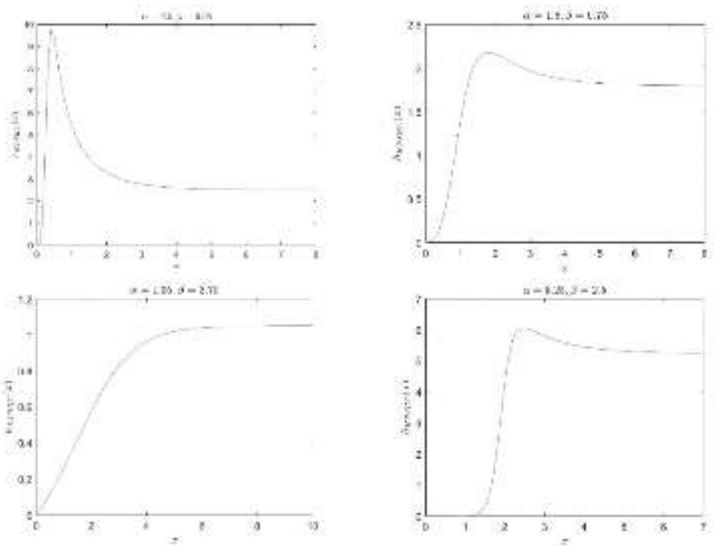

Figure 7. The plots of hazard rate function of KPSHC(α, β) for $\alpha > 1$ and $\beta > 0$.

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Therefore, based on the results from all considered cases, the KPSHC distribution exhibits potential for effectively modeling and generating data with monotone (monotonically increasing) and non-monotone (bathtub and unimodal) hazard rates.

F. Flexibility of Kafi-Pawat Logarithmic Distribution

Figures According to Section 2, the hazard rate function (HRF) of Kafi-Pawat logarithmic (KPL) distribution is given as follows:

$$h_{\text{KPL}}(x) = \frac{\frac{\alpha}{x+1}\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha-1}}{\beta\left[1+\left(\frac{\ln(x+1)}{\beta}\right)^{\alpha}\right]},\ x > 0. \qquad (14)$$

The HRF of the KPL distribution, as defined in Equation (14), exhibits various forms depending on the values of its parameters. In this distribution, both α and β are shape parameters. The analysis of the KPSE hazard rate function is presented on a case-by-case basis as follows:

Case 1: $0 < \alpha \leq 1$ and $\beta > 0$. In this case, the HRF given in Equation (14) is a decreasing function. However, since Equation (14) is too complex, a graphical approach becomes relevant to be used to show the curves produced by this HRF. Figure 8 presents the plots of the hazard rate function (HRF) for KPL(α, β) when $0 < \alpha \leq 1$.

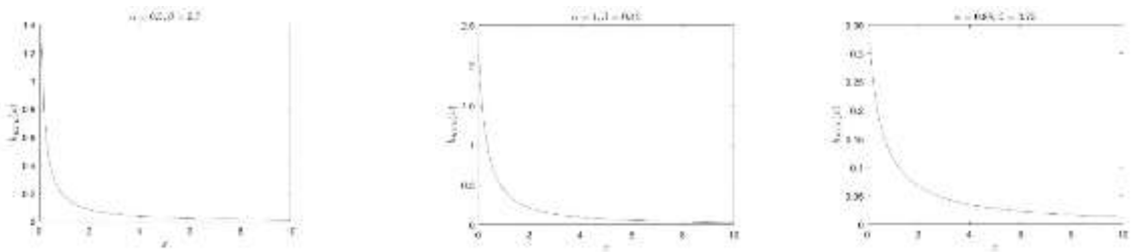

Figure 8. The hazard rate of KPL(α, β) for $0 < \alpha \leq 1$.

Case 2: $\alpha > 1$ and $\beta > 0$. In this case, the HRF given in Equation (14) is a unimodal function. However, since Equation (14) is too complex, a graphical approach becomes relevant to be used to show the curves produced by this HRF. Figure 9 presents the plots of the hazard rate function (HRF) for KPL(α, β) when $\alpha > 1$.

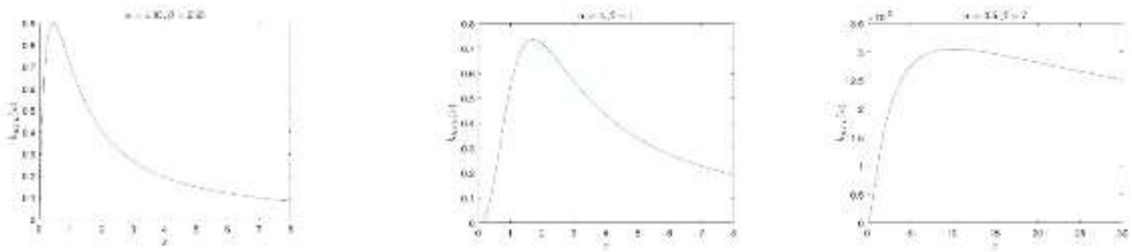

Figure 9. The hazard rate of KPL(α, β) for $\alpha > 1$.

Therefore, based on the results from all considered cases, the KPL distribution exhibits potential for effectively modeling and generating data with monotone (decreasing) and non-monotone (unimodal) hazard rates.

BASIC DISTRIBUTIONAL QUANTITIES OF KAFI-PAWAT FAMILY

G. Mean of Kafi-Pawat Family of Distribution

Proposition 2 formally states the expression for the mean (first moment) of the KP family.

Proposition 2. Let X follows a distribution in KP family with parameter $\alpha > 0$ and $\beta > 0$. The mean of X is

$$E(X) = \int_0^{\infty} \frac{1}{1+\left(\frac{f(x)}{\beta}\right)^{\alpha}}\,dx. \qquad (15)$$

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Proof. The mean of X can be obtained as follows:

$$E(X) = \int_0^\infty S_{KP}(x)\,dx = \int_0^\infty \frac{1}{1+\left(\frac{f(x)}{\beta}\right)^\alpha}dx \qquad \blacksquare$$

By Proposition 2, we obtain:

- The mean of inverse Burr distribution is $E(X) = \int_0^\infty \frac{1}{1+\left(\frac{x}{\beta}\right)^\alpha}dx = \frac{\beta}{\alpha}\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(1-\frac{1}{\alpha}\right)$ for $\alpha > 1$.
- The mean of KPL distribution is $E(X) = \int_0^\infty \frac{1}{1+\left(\frac{\ln(x+1)}{\beta}\right)^\alpha}dx = \infty$.
- The mean of KPSE distribution is $E(X) = \int_0^\infty \frac{1}{1+\left(\frac{\exp(x)-1}{\beta}\right)^\alpha}dx$.
  However, mathematical software, such as WolframAlpha, is required to solve its mean.
- The mean of KPSHC distribution is $E(X) = \int_0^\infty \frac{1}{1+\left(\frac{\cosh(x)-1}{\beta}\right)^\alpha}dx$.
  However, mathematical software, such as WolframAlpha, is required to solve its mean.

H. *Quantile Function of Kafi-Pawat Family of Distribution*

As the CDF of the KP family possesses a closed-form expression, the corresponding quantiles can be derived analytically. This property facilitates the direct computation of the *r* quantile (100*r*-th percentile) without requiring numerical approximation methods. Proposition 3 formally states the expression for the *r* quantile of the KP family.

Proposition 3. Let X follows a distribution in KP family with parameter $\alpha > 0$ and $\beta > 0$. The *r* quantile (100*r*-th percentile) of X is

$$Q(r) = x_r = f^{-1}\left(\beta\left(\frac{r}{1-r}\right)^{\frac{1}{\alpha}}\right), \qquad (16)$$

where $0 < r < 1$. In particular, if $r = 0.5$, the 50-th percentile (the median) of X is

$$Q(0.5) = x_{0.5} = f^{-1}(\beta), \qquad (17)$$

Proof. The *r* quantile of X can be obtained as follows:

$$F_X(x_r) = r \Leftrightarrow \frac{\left(\frac{f(x_r)}{\beta}\right)^\alpha}{1+\left(\frac{f(x_r)}{\beta}\right)^\alpha} = r \Leftrightarrow \left(\frac{f(x_r)}{\beta}\right)^\alpha(1-r) = r \Leftrightarrow f(x_r) = \beta\left(\frac{r}{1-r}\right)^{\frac{1}{\alpha}} \Leftrightarrow x_r = f^{-1}\left(\beta\left(\frac{r}{1-r}\right)^{\frac{1}{\alpha}}\right)$$

The median of X is

$$x_{0.5} = f^{-1}\left(\beta\left(\frac{0.5}{1-0.5}\right)^{\frac{1}{\alpha}}\right) = f^{-1}(\beta) \qquad \blacksquare$$

By Proposition 3, we obtain:

- The *r* quantile of inverse Burr distribution is $x_r = \beta\left(\frac{r}{1-r}\right)^{\frac{1}{\alpha}}$.
- The *r* quantile of KPSE distribution is $x_r = \ln\left\{\beta\left(\frac{r}{1-r}\right)^{\frac{1}{\alpha}} + 1\right\}$.
- The *r* quantile of KPSHC distribution is $x_r = \operatorname{arccosh}\left\{\beta\left(\frac{r}{1-r}\right)^{\frac{1}{\alpha}} + 1\right\}$.

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- The r quantile of KPL distribution is $x_r = \exp\left\{\beta\left(\frac{r}{1-r}\right)^{\frac{1}{\alpha}} + 1\right\} - 1$.

PARAMETER ESTIMATION

In this study, the Maximum Likelihood Estimation (MLE) method is employed to estimate the parameters of all distributions in the Kafi-Pawat (KP) family. MLE is chosen due to its desirable asymptotic properties, particularly the fact that it yields consistent and asymptotically unbiased estimators as the sample size increases. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n drawn independently and identically distributed (i.i.d.) from a distribution in the KP family with PDF

$$f_{\text{KP}}(x; \alpha, \beta) = \frac{\alpha\left(f(x)\right)^{\alpha-1} f'(x)\beta^\alpha}{\left[\left(f(x)\right)^\alpha + \beta^\alpha\right]^2}, \qquad x > 0,$$

where $\alpha > 0$, $\beta > 0$, and $f(x)$ is a positive, non-decreasing, continuous function over $x > 0$. Let $x_1, x_2, \ldots, x_n$ represent the corresponding observed values of the random variables $X_1, X_2, \ldots, X_n$, respectively. The likelihood function of this distribution, based on this sample, is derived as follows:

$$L(\alpha, \beta; x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} f_{\text{KP}}(x_i; \alpha, \beta) = \prod_{i=1}^{n} \frac{\alpha\left(f(x_i)\right)^{\alpha-1} f'(x_i)\beta^\alpha}{\left[\left(f(x_i)\right)^\alpha + \beta^\alpha\right]^2}$$

$$= (\alpha\beta^\alpha)^n \left(\prod_{i=1}^{n} f(x_i)\right)^{\alpha-1} \left(\prod_{i=1}^{n} f'(x_i)\right) \left(\prod_{i=1}^{n} \left[\left(f(x_i)\right)^\alpha + \beta^\alpha\right]\right)^{-2}.$$

This leads to the log-likelihood function as follows:

$$\ell(\alpha, \beta; x_1, x_2, \ldots, x_n) = n(\ln(\alpha) + \alpha \ln \beta) + (\alpha - 1)\sum_{i=1}^{n} \ln f(x_i)$$
$$+ \sum_{i=1}^{n} \ln f'(x_i) - 2\sum_{i=1}^{n} \ln\left[\left(f(x_i)\right)^\alpha + \beta^\alpha\right]. \qquad (18)$$

By taking the first derivative of Equation (18) with respect to α and β, and equating to zero, the following nonlinear equations are obtained.

$$\frac{\partial \ell}{\partial \alpha} = \frac{n}{\alpha} + n \ln \beta + \sum_{i=1}^{n} \ln f(x_i) - 2\sum_{i=1}^{n} \frac{\beta^\alpha \ln(\beta) + \left(f(x_i)\right)^\alpha \ln\left(f(x_i)\right)}{\left(f(x_i)\right)^\alpha + \beta^\alpha} = 0 \qquad (19)$$

$$\frac{\partial \ell}{\partial \beta} = \frac{n\alpha}{\beta} - 2\alpha\beta^{\alpha-1} \sum_{i=1}^{n} \frac{1}{\left(f(x_i)\right)^\alpha + \beta^\alpha} = 0 \qquad (20)$$

Equations (19) and (20) constitute a nonlinear system of equations that cannot be solved analytically to obtain closed-form solutions for the parameters α and β. As a result, a numerical optimization technique is required to compute the Maximum Likelihood Estimates (MLEs) of these parameters. In this study, the Newton-Raphson method is employed to iteratively approximate the estimates of α and β, due to its efficiency and rapid convergence properties when applied to differentiable functions.

APPLICATIONS TO REAL DATASETS

This section applies all the established distributions to four real-world datasets to assess its empirical performance and demonstrate its applicability in modeling non-negative continuous data. The goodness-of-fit of each considered distribution to the respective datasets is assessed using several score-based statistical criteria. Specifically, the evaluation is based on the log-likelihood values, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and the Kolmogorov-Smirnov (KS) goodness-of-fit statistic, along with the corresponding p-values. These measures collectively provide a comprehensive framework for determining which distribution offers the best fit to the observed data, with lowest AIC and BIC values indicating the best fit amongst competing models.

I. COVID-19 Dataset

The first dataset consists of daily COVID-19 mortality in the United States over a period of 102 days, spanning from 28 March to 7 July 2020 [19]. Table 1 summarizes the fitting results for the COVID-19 dataset, and it indicates that all considered distributions offer a good fit, as evidenced by the p-value of the KS test exceeding the significance level of 0.05, suggesting no statistically

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significant difference between the empirical and fitted distributions. Furthermore, the inverse Burr distribution achieves the lowest values for both AIC and BIC among all competing models, thereby identifying it as the most appropriate model for representing this dataset.

Table 1. Fitting result for COVID-19 mortality rate in the US from 28 March to 7 July 2020.

Distribution	KS statistic (p-value)	Log-likelihood	AIC	BIC	Fitted parameters
KPSE	0.094232 (0.3254)	220.3964	-436.7928	-431.5428	$\hat{\alpha} = 2.43254$ $\hat{\beta} = 0.04229$
KPSHC	0.095155 (0.3142)	219.993	-435.986	-430.7361	$\hat{\alpha} = 1.24116$ $\hat{\beta} = 0.00085$
KPL	0.096044 (0.3036)	219.5737	-435.1475	-429.8975	$\hat{\alpha} = 2.53230$ $\hat{\beta} = 0.04041$
Inverse Burr	0.09025 (0.3771)	223.3872	-442.7744	-437.5245	$\hat{\alpha} = 2.14763$ $\hat{\beta} = 0.06339$

J. Environmental Dataset

The second dataset contains yearly measurements of snow accumulation (in inches) taken at Raleigh-Durham Airport in North Carolina, spanning the years 1948 to 2000 [20]. Table 2 summarizes the fitting results for the snow accumulation dataset, and it indicates that all considered distributions offer a good fit, as evidenced by the p-value of the KS test exceeding the significance level of 0.05, suggesting no statistically significant difference between the empirical and fitted distributions. The inverse Burr distribution achieves the lowest values for both AIC and BIC among all competing models, thereby identifying it as the most appropriate model for representing this dataset.

Table 2. Fitting result for the snow accumulation (in inches) in the Raleigh-Durham airport.

Distribution	KS statistic (p-value)	Log-likelihood	AIC	BIC	Fitted parameters
KPSE	0.10507 (0.4899)	-116.192	236.3839	240.6702	$\hat{\alpha} = 2.43254$ $\hat{\beta} = 0.04229$
KPSHC	0.10731 (0.4626)	-111.4267	226.8534	231.1396	$\hat{\alpha} = 0.54625$ $\hat{\beta} = 0.94608$
KPL	0.13951 (0.1721)	-112.7584	229.5169	233.8031	$\hat{\alpha} = 2.1072$ $\hat{\beta} = 0.7523$
Inverse Burr	0.11586 (0.3662)	-108.2204	220.4407	224.727	$\hat{\alpha} = 1.4921$ $\hat{\beta} = 1.1970$

K. Cancer Remission Dataset

The third dataset comprises the remission times (in months) of 128 bladder cancer patients [21]. Table 3 summarizes the fitting results for dataset 3, and it indicates that all considered distributions, except the KPSE, offer a good fit, as evidenced by the p-value of the KS test exceeding the significance level of 0.05, suggesting no statistically significant difference between the empirical and fitted distributions. Furthermore, among all competing distributions, the inverse Burr distribution yields the lowest values for both AIC and BIC for dataset 3. It indicates that inverse Burr is the most suitable model for this dataset.

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Table 3. Fitting result for the remission times (in months) of 128 bladder cancer patients

Distribution	KS statistic (p-value)	Log-likelihood	AIC	BIC	Fitted parameters
KPSE	0.12853 (0.02913)	-445.9701	895.9402	901.6442	$\hat{\alpha} = 0.2194$ $\hat{\beta} = 1857$
KPSHC	0.11806 (0.05643)	-440.9633	885.9265	891.6306	$\hat{\alpha} = 0.21575$ $\hat{\beta} = 882.0287$
KPL	0.071733 (0.5254)	-424.5409	853.0818	858.7859	$\hat{\alpha} = 3.62434$ $\hat{\beta} = 1.90166$
Inverse Burr	0.039888 (0.987)	-411.4575	826.9151	832.6191	$\hat{\alpha} = 1.7252$ $\hat{\beta} = 6.0898$

L. Labor Economy Dataset

The fourth dataset includes the average monthly net wage or salary (in million Rupiahs) earned by employees from their primary job across 17 sectors in Indonesia (source: https://www.bps.go.id/en/statistics-table/2/MTUyMSMy/rata-rata-upah-gaji.html). Table 4 presents the fitting results for dataset 4, and it indicates that all considered distributions offer a good fit, as evidenced by the p-value of the KS test exceeding the significance level of 0.05, implying no statistically significant discrepancy between the empirical data and the fitted model. Furthermore, among all competing distributions, the KPSHC distribution yields the lowest values for both AIC and BIC for dataset 4. It indicates that KPSHC is the most suitable model for this dataset.

Table 4. Fitting result for the average net wage/salary (in million Rupiahs) per month

Distribution	KS statistic (p-value)	Log-likelihood	AIC	BIC	Fitted parameters
KPSE	0.12909 (0.9056)	-24.98101	53.96202	55.62844	$\hat{\alpha} = 1.5609$ $\hat{\beta} = 28.8902$
KPSHC	0.12653 (0.9171)	-24.83753	53.67506	55.34149	$\hat{\alpha} = 1.5013$ $\hat{\beta} = 13.8069$
KPL	0.10229 (0.9861)	-25.08974	54.17948	55.84591	$\hat{\alpha} = 10.31748$ $\hat{\beta} = 1.45669$
Inverse Burr	0.10997 (0.9719)	-24.92882	53.85763	55.52406	$\hat{\alpha} = 5.4562$ $\hat{\beta} = 3.3178$

CONCLUSIONS

This research presents the novel Kafi-Pawat (KP) family of distributions. The distributional functions the KP family have been thoroughly derived. To understand the flexibility of the proposed family, some members of this family are established and discussed, that is, the inverse-Burr, the Kafi-Pawat shifted exponential, the Kafi-Pawat shifted hyperbolic cosine, and the Kafi-Pawat logarithmic distributions. Notably, the hazard rate functions of considered distributions can produce both monotone and non-monotone shapes which confirm that the KP family is flexible.

Parameter estimation for all distribution in KP family was carried out using the maximum likelihood estimation (MLE) method. Due to the difficulty in finding closed-form solutions, a numerical method is required to compute the estimators. We then present the application of all considered distributions to real datasets and perform a comparative study among them. The Kolmogorov-Smirnov goodness-of-fit test indicates that all distributions, except the Kafi-Pawat shifted exponential, provide a good fit for all real datasets, with statistically significant p-values supporting their adequacy. Moreover, for datasets 1-3, the inverse Burr distribution achieves the lowest AIC and BIC, thus representing the most suitable model for these datasets. In contrast, for dataset 4, the Kafi-Pawat shifted hyperbolic cosine distribution attains the lowest AIC and BIC, making it the most appropriate model for this dataset. Nevertheless, other members of the Kafi-Pawat family can be considered in future studies.

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