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Pseudocontractive Mappings: A Review of New Iteration Processes and Convergence Results

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ABSTRACT: Pseudocontractive mappings are a fundamental concept in nonlinear analysis, with wide-ranging applications in optimization, differential equations, and control theory. This review provides a comprehensive overview of recent iteration processes for approximating fixed points of pseudocontractive mappings, highlighting theoretical advancements, convergence results, and numerical implementations. Various iterative schemes are examined in terms of their mathematical foundations, convergence properties, and computational effectiveness. The paper consolidates the existing literature, identifies open research problems, and outlines potential directions for future investigations.

KEYWORDS: Banach spaces, Convergence analysis, Fixed point theory, Iteration processes, Nonlinear analysis, Iterative methods, Pseudocontractive mappings, Strong convergence, Weak convergence.

INTRODUCTION

Pseudocontractive mappings occupy a central role in nonlinear analysis, with significant applications in optimization, nonlinear differential and integral equations, and control theory [Petruşel& Yao, 2011; Chang & Kim, 2013]. The study of fixed points of such mappings has been a subject of extensive research due to its theoretical depth and practical utility.

Classical iterative schemes such as the Mann iteration [Mann, 1953], Ishikawa iteration [Ishikawa, 1974], and Halpern iteration [Halpern, 1967] have laid the foundation for fixed point approximation. However, recent research has proposed modified, hybrid, and inertial-type iterations aimed at accelerating convergence and enhancing numerical stability [Ceng& Yao, 2010; Kim & Xu, 2017; Saleem et al., 2022].

This review surveys these developments, focusing on novel iteration processes and their convergence properties. By examining recent contributions, we provide a state-of-the-art synthesis, identify current limitations, and propose promising directions for further research.

APPLICATIONS OF PSEUDOCONTRACTIVE MAPPINGS

1. Optimization**:** Iterative schemes for convex optimization and variational inequalities often rely on pseudocontractive frameworks [Yao &Shahzad, 2015].
2. Differential Equations**:** Fixed point results for pseudocontractive mappings support the existence and uniqueness of solutions to nonlinear differential and integral equations [Okeke et al., 2025].
3. Control Theory**:** Applied in system stability analysis and optimal control design, offering robust modeling tools [Debnath et al., 2021].

IMPORTANCE OF FIXED POINT THEORY

Fixed point theory underpins the analysis of pseudocontractive mappings by enabling structural characterizations and convergence investigations. Iterative methods not only generalize classical fixed point results but also yield constructive algorithms for applications in applied mathematics and computational sciences [Zhou et al., 2022; Kalsoom et al., 2021].

ITERATIVE METHODS

Classical iterative schemes have played a central role in fixed point theory. The Mann iteration, introduced by Mann (1953), is considered a fundamental averaging process and laid the foundation for subsequent developments. Building upon this, Ishikawa

(1974) proposed a refinement of the Mann scheme that ensures stronger convergence properties. Later, Halpern (1967) developed another iterative method that is particularly notable for guaranteeing strong convergence under appropriate conditions.

In recent years, several new iterative processes have been introduced to address the limitations of these classical approaches. For example, Kim and Xu (2017) proposed a modified version of the Mann iteration, which improves convergence even under weaker assumptions. Ceng and Petruşel (2016) designed hybrid iteration schemes that combine different strategies to enhance both stability and efficiency. More recently, Saleem et al. (2022) introduced inertial-type iterations, which incorporate momentum terms to accelerate convergence while maintaining robustness.

METHODOLOGY

This review employed a systematic four-stage methodology:

1. Literature Review: Search in MathSciNet, Scopus, and Web of Science using targeted keywords. Priority was given to works from the past decade, alongside foundational studies [Mann, 1953; Ishikawa, 1974].
2. Classification and Organization: Iterative schemes categorized into classical (Mann, Ishikawa, Halpern) and modern (modified, hybrid, inertial).
3. Critical Evaluation: Assessment of proof techniques, assumptions, and numerical experiments [Ceng& Yao, 2010; Qin & Cho, 2014].
4. Synthesis and Conclusion: Integration of insights to highlight emerging trends and future directions.

RESULTS AND DISCUSSION

New Iteration Processes

In recent years, several novel iteration processes have been developed to overcome the shortcomings of traditional schemes. Kim and Xu (2017) introduced the **modified Mann iteration, which has been shown to provide improved stability and convergence results in Banach spaces. Similarly, Ceng and Petruşel (2016) proposed hybrid schemes that effectively combine classical strategies, thereby yielding more robust and reliable performance. More recently, Saleem et al. (2022) advanced the study of inertial-type methods, which incorporate momentum-based terms to accelerate convergence and have proven particularly effective in large-scale optimization problems.

Convergence Results

The convergence behavior of these iterative processes has attracted considerable attention. Yao et al. (2017) established strong convergence results for both modified Mann and hybrid iteration schemes, thereby ensuring reliability in practical applications. On the other hand, weak convergence results, as demonstrated by Qin and Cho (2014), extend applicability to broader classes of mappings, offering general yet valuable insights into the underlying structures of iterative methods.

Numerical Implementations

Beyond theoretical developments, numerical experiments have validated the practical utility of these schemes. Empirical studies, such as those by Okeke et al. (2025), confirm the effectiveness of the new iterative processes in diverse applications, including optimization problems, equilibrium models, and fractional differential equations.

4. Comparative Analysis

Sr.No.	Iteration Process	Convergence Rate	Stability
1.	Modified Mann Iteration	Faster	Improved
2.	Hybrid Iteration Schemes	Comparable	Enhanced
3.	Inertial-Type Iterative Methods	Accelerated	Robust

This comparison highlights the advantage of inertial methods, which balance speed with stability.

Future Directions

Looking ahead, several promising research avenues can be identified in the study of iterative methods for pseudocontractive mappings. One important direction concerns the precise analysis of convergence rates for specialized classes of pseudocontractive mappings, which would not only strengthen the theoretical framework but also provide deeper insights into the efficiency of existing schemes. Another significant line of investigation lies in exploring applications to large-scale optimization problems and high-dimensional control systems, where the scalability and robustness of iterative methods are of critical importance. Furthermore, the development of hybrid algorithms that integrate multiple iterative strategies represents an exciting frontier, offering the potential to combine the strengths of different approaches to achieve improved stability, faster convergence, and broader applicability.

CONCLUSION

This review has highlighted recent developments in the study of pseudocontractive mappings, with particular attention to the introduction of new iterative processes and their convergence properties. Advances such as the modified Mann, hybrid, and inertial-type iterations have demonstrated notable improvements in both convergence speed and numerical stability. Convergence theorems, encompassing both strong and weak forms, have further enriched the theoretical understanding of these methods. Importantly, empirical implementations confirm the effectiveness of these schemes across a wide range of applied problems.

From these findings, several key insights emerge. Pseudocontractive mappings continue to serve as a cornerstone of nonlinear analysis, while newly developed iterative schemes significantly enhance convergence behavior. At the same time, substantial opportunities remain for extending their applications and refining the theoretical framework that underpins them. Looking ahead, future research should aim to provide a deeper analysis of convergence rates for specialized classes, broaden the scope of applications in optimization, equilibrium, and control, and pursue the design of unified algorithms that integrate the strengths of both classical and modern iterations.

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