International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

DSGE Model with Structure Variance

Cristian Andres Cruz Torres
Department of Statistics, National Autonomous University of Honduras

ABSTRACT: In this paper we develop full Bayesian inference for a rich class of DSGE models (Dynamics Stochastic General Equilibrium). It is well known that any DSGE model after being log-linearized could be written as a state space model with Gaussian shocks in the state equation, in this context we propose to explore a DSGE with structure variance on the shocks, building a normal model with stochastic volatility and a Student-t model with stochastic volatility. The methodologies are applied to real data and is compared with the traditional approach.

KEYWORDS: Bayesian inference, DSGE models, state-space model, stochastic volatility, structure variance.

I. INTRODUCTION

Dynamic Stochastic General Equilibrium – DSGE, are macroeconomic models designed to explain the behavior of business cycles. Their distinguishing feature is that their theoretical framework is derived from microeconomic foundations. These models typically assume that agents optimize their choices, leading to rational expectations in their decision-making. Furthermore, each agent maximizes a utility function, subject to agent-specific constraints.

These models generally incorporate a representative household and a set of firms. Households consume goods, supply labor, and accumulate capital to maximize their utility function under various constraints (e.g., budget constraints, labor demand constraints, capital motion laws, etc.). Firms, on the other hand, produce goods, hire labor, and maximize profits subject to their respective constraints (e.g., demand constraints). DSGE models are then augmented by a monetary authority's reaction function.

The objective of this article is to examine log-linearized DSGE models expressed in state-space form while incorporating heteroskedastic shocks. We consider shocks with structured variance, exploring both normal and Student-t distributions with stochastic volatility.

II. DSGE MODELS

A DSGE model comprises the following agents: a firm producing a final good, a continuum of intermediate goods produced by monopolistically competitive firms, a representative household, and a government acting as both fiscal and monetary authority.

To describe the model, it is necessary to explicitly specify the economic environment: preferences, technology, resources, and information. Additionally, we must define the model's objective. Typically, DSGE models consist of three key components: The economic environment in which decision-makers operate; the decision rules governing their behaviour; and the uncertainty they face when making decisions.

Collectively, these components form a nonlinear system of expectational difference equations. Such systems generally lack closed-form analytical solutions in empirical analyses but can be rendered computationally tractable through a systematic solution process, detailed in this section.

The solution process involves three main steps:

1. Linear Approximation: The model's nonlinear equations are approximated linearly using first-order Taylor expansions, resulting in a system of expectational difference equations.
2. Solving the Linearized System: The resulting linear system is solved, expressing variables as deviations from their steady-state values. This solution is then ready for implementation.
3. Model Estimation: The system is cast into a state-space representation. Assuming normally distributed shocks, the model's likelihood can be evaluated using the Kalman filter, and its parameters can be estimated via maximum likelihood or Bayesian methods.

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

A. Equilibrium Equations

It can be highly useful to classify time-dependent variables into two distinct sets: state variables and control variables. In period t, state variables are those whose values are determined either by past actions or by exogenous processes (e.g., stochastic shocks). Control variables, on the other hand, are explicitly chosen to optimize a given objective function. Often, the modeler has some flexibility in defining which variables are treated as state or control variables.

Once state and control variables are defined, the solution procedure follows these steps:

1. Derive the equilibrium conditions, including constraints, first-order conditions, and market-clearing conditions.
2. Compute the steady-state values of state and control variables.
3. Log-linearize the equilibrium equations to obtain an approximate linear system in terms of log-deviations from the steady state.
4. Solve the system recursively using a rational expectations method.
5. Express the resulting system in state-space form.
6. Estimate the model parameters using an appropriate numerical algorithm.

To derive equilibrium conditions, dynamic programming techniques are typically employed to solve the optimization problems of agents. The steady state is obtained by assuming no stochastic shocks and solving for the long-run equilibrium values of all variables. Given the inherent nonlinearity of DSGE models, a standard approach is to approximate the system using a first-order Taylor expansion around the steady state. This linearization relies on the assumption that if the model remains close to its steady state, the linearized version provides a reasonable approximation of the original system. The method involves:

- Applying a Taylor series expansion to each equilibrium equation.
- Expressing variables as logarithmic deviations from their steady-state values.
- Substituting these approximations into the original system to obtain a linearized structure.

B. Solving the model

Once the model is log-linearized, the resulting system often includes forward-looking equations involving expectations of future variables, that is, it becomes a linear system of expectational difference equations. Before expressing the model in its final approximated form, these expectations must be resolved. It can be shown that, after log-linearization and appropriate simplifications, the system of equilibrium equations can be expressed in the following general form:

$$A\begin{bmatrix}\hat{x}_{t+1}\\ E_t\hat{y}_{t+1}\end{bmatrix} = B\begin{bmatrix}\hat{x}_t\\ \hat{y}_t\end{bmatrix} + C\hat{v}_{t+1}, \qquad (1)$$

Where $\hat{x}_t$ be the $n \times 1$ vector of log-linearized endogenous state variables at time t, z_t be the vector of exogenous state variables following an autoregressive process, $\hat{y}_t$ be the $m \times 1$ vector of log-linearized control variables at time t and v_t be the $n_v \times 1$ vector of exogenous shocks. The matrices A, B, and C are functions of the structural parameters from the optimization problem. The log-linearized system (1) contains a conditional expectation of future variables, which must be resolved before expressing the system in state-space form. The following methods are commonly used to solve forward-looking rational expectations systems: Blanchard-Kahn method (Blanchard and Kahn, 1980), Klein´s method (Klein, 2000) and Method of Undetermined Coefficients (Uhlig, 2001).

After applying any of these solution methods, the system can be expressed in state-space form as:

$$S_{t+1} = GS_t + Fv_{t+1}$$

Where the matrices G and F depends on the parameters. Let Y_t the set of observable variables in a period t. Therefore, the state-space model has the form:

$$Y_t = a + HS_t,$$
$$S_{t+1} = GS_t + Fv_{t+1} \qquad (2)$$

where a and H are matrices of known constants or functions of the parameters. In the literature, it is assumed that $v_{t+1} \sim N(0, \Sigma)$ has a multivariate normal distribution with diagonal dispersion matrix $\Sigma = diag(\sigma_1^2, \dots, \sigma_{n_v}^2)$.

Given that the above system is written in state space and normal shocks are assumed in equation (2), we can use the Kalman filter to evaluate the likelihood conditioned on the parameters

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

III. DSGE MODELS WITH STRUCTURE VARIANCE

A. *Stochastic Volatility Model*

A univariate stochastic volatility model as defined in Chib et al. (2009) and Asai et al. (2006) is given by:

$$y_t = \exp\left(\frac{h_t}{2}\right)\epsilon_t, \qquad t = 1, \dots, n,$$
$$h_t = \mu + \phi(h_{t-1} - \mu) + \eta_t, \qquad t = 2, \dots, n,$$
$$h_1 \sim N\left(\mu, \frac{\sigma_\eta^2}{1-\phi^2}\right),$$
$$\begin{pmatrix}\epsilon_t\\ \eta_t\end{pmatrix} | h_t \sim N_2(0, \Sigma), \qquad \Sigma = \begin{pmatrix}1 & 0\\ 0 & \sigma_\eta^2\end{pmatrix},$$

where y_t is univariate, h_t is the univariate latent variable and $N(0,\sigma^2)$ and $N_m(0,\Sigma)$ denote the univariate normal distribution with mean 0 and variance σ^2 and the multivariate distribution of dimension m with mean 0 and covariance matrix Σ respectively. In this model, conditioned on the parameters $(\mu, \phi, \sigma_\eta^2)$, the first equation represents the distribution of y_t conditioned on h_t and the second equation represents the Markovian evolution of h_t given h_{t-1}. This stochastic volatility model is therefore a state-space model with a linear evolution in the state variable h_t, but with a nonlinear observation equation. Note that $var(y_t|h_t) = exp(h_t)$, therefore, h_t can be understood as the logarithm of the conditional variance at each time *t*. To ensure that the evolution of these log-volatilities is stationary, it is assumed that $|\phi| < 1$.

In the multivariate context, let the observation vector be $\boldsymbol{y_t} = (y_{1t}, \dots, y_{pt})'$. Our objective is to model the time-varying conditional covariance matrix of $\boldsymbol{y}_t$ While numerous specifications exist (see, e.g., Asai et al., 2006; Jacquier et al., 2004; Yu & Meyer, 2006 for bivariate applications), we begin with the simplest generalization of the univariate stochastic volatility (SV) model:

$$\boldsymbol{y_t} = V_t^{\frac{1}{2}}\epsilon_t, \ \ t = 1, \dots, n,$$
$$h_t = \mu + \phi(h_{t-1} - \mu) + \eta_t, \qquad t = 2, \dots, n,$$
$$h_1 \sim N_p(\mu, \Sigma_0),$$

with $V_t^{1/2} = diag\left(\exp\left(\frac{h_{1t}}{2}\right), \dots, \exp\left(\frac{h_{pt}}{2}\right)\right)$ and $\begin{pmatrix}\epsilon_t\\ \eta_t\end{pmatrix} | h_t \sim N_{2p}(0, \Sigma),\ \Sigma = \begin{pmatrix}\Sigma_\epsilon & 0\\ 0 & \Sigma_{\eta\eta}\end{pmatrix},$

Where $h_t = (h_{1t}, \dots, h_{pt})$. To reduce the computational cost, especially when p is large, the log-volatilities can be assumed conditionally independent. In this case:

$$\Phi = diag(\phi_{11}, \dots, \phi_{pp}) \ \text{ and } \ \Sigma_{\eta\eta} = diag(\sigma_{1,\eta\eta}^2, \dots, \sigma_{p,\eta\eta}^2),$$

are both diagonal matrices. For identification purposes, the diagonal elements of Σ_ϵ must be ones, which means that $\Sigma_\epsilon = R$ is the correlation matrix of ϵ_t. Therefore, conditioned on h_t, $var(y_t) = V_t^{1/2} R V_t^{1/2}$ is time-varying, but the correlation matrix *R* is not time-varying.

Since we are considering Φ as diagonal, this implies that we are not allowing Granger causality effects between volatilities; furthermore, by considering $\Sigma_{\eta\eta}$ diagonal, we are not considering cross-dependence between volatilities. Finally, let us note that $cov(\epsilon_t, \eta_t|h_t) = 0$, which means that no leverage effects are considered between the observational and volatility shocks.

B. *DSGE with Stochastic Volatility*

Remembering that every log-linearized DSGE model can be written in state space, consider the equation of state given by:

$$S_{t+1} = GS_t + Fv_{t+1}$$

In order to introduce stochastic volatility, let us assume that, for each time *t*, $v_t \sim N(0, \Sigma_t)$ with $\Sigma_t = V_t^{1/2} R V_t^{1/2}$, which is analogous to the decomposition proposed by Barnard et al. (2000). At present time *t*, the covariance matrix varies in time and $V_t^{\frac{1}{2}}$ is defined by $V_t^{1/2} = diag\left(\exp\left(\frac{h_{1t}}{2}\right), \dots, \exp\left(\frac{h_{pt}}{2}\right)\right)$, which also corresponds to a diagonal matrix whose elements are the standard deviations that vary in time. In the DSGE literature, there are some works that incorporate a stochastic volatility structure. For example, Justiniano and Primiceri (2008) and Fernandez-Villaverde et al. (2015) make applications of stochastic volatility with an

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

independent structure. It is important to emphasize that they consider the case when $R = I$, the identity matrix. Our contribution is to consider the correlation matrix differently from the identity.

Therefore, the complete model can be written as:

$$
\begin{aligned}
Y_t &= a + HS_t,\\
S_{t+1} &= GS_t + Fv_{t+1},\\
h_t &= \mu + \phi(h_{t-1} - \mu) + \eta_t, \qquad t = 2, \dots, n,\\
h_1 &\sim N_p(\mu, \Sigma_0)
\end{aligned}
$$

with $\eta_t | h_t \sim N_p(0, \Sigma_{\eta\eta})$, Φ and $\Sigma_{\eta\eta}$ diagonal matrices. The matrices a, H, G and F are matrices that depend on the structural parameters of the DSGE. Note that this system is linear with respect to the observations Y_t and the state variable h_t, but is nonlinear with respect to S_t.

C. ***DSGE Student-t with Stochastic Volatility***

We extend the standard DSGE framework by introducing heavy-tailed innovations with time-varying volatility and correlated shocks. Unlike Curdia et al. (2014), who assume independent t-Student shocks, our contribution explicitly models cross-shock dependence, capturing tail co-movements often observed in macroeconomic and financial data.

$$
\begin{aligned}
Y_t &= a + HS_t,\\
S_{t+1} &= GS_t + Fv_{t+1},\\
v_t &\sim N(0, \lambda_t^{-1}\Sigma_t), \qquad \lambda_t \sim Ga\left(\frac{\nu}{2}, \frac{\nu}{2}\right), \Sigma_t = V_t^{\frac{1}{2}} R V_t^{\frac{1}{2}},\\
V_t^{\frac{1}{2}} &= diag\left(\exp\left(\frac{h_{1t}}{2}\right), \dots, \exp\left(\frac{h_{pt}}{2}\right)\right),\\
h_t &= \mu + \phi(h_{t-1} - \mu) + \eta_t, \qquad t = 2, \dots, n,\\
h_1 &\sim N_p(\mu, \Sigma_0)
\end{aligned}
$$

With $\eta_t | h_t \sim N_p(0, \Sigma_{\eta\eta})$, Φ and $\Sigma_{\eta\eta}$ are diagonal matrices. The matrices a, H, G and F are matrices that depend on the structural parameters of the DSGE. Note that when we integrate the parameters $\lambda_t, \forall t$, we realize that S_t has a Student-t distribution with mean GS_{t-1}, scatter matrix Σ_t and ν degrees of freedom, that is, $S_t \sim t_\nu(GS_{t-1}, \Sigma_t)$.

IV. EXAMPLE

We adapt the framework from Chib & Ramamurthy (2014) to a two-dimensional DSGE model with stochastic volatility (SV) in one shock, while keeping the second shock homoscedastic. Given that DSGE models typically use low-frequency data (e.g., quarterly macroeconomic series), we simulate 100 observations to reflect real-world estimation challenges. Consider the following DSGE model written in state space:

$$
\begin{aligned}
y_t &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix},\\
\begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix} &= \begin{bmatrix} g_{1,1} & g_{1,2} \\ 0 & g_{2,2} \end{bmatrix} \begin{bmatrix} s_{1,t-1} \\ s_{2,t-1} \end{bmatrix} + \begin{bmatrix} \exp\left(\frac{h_{1,t}}{2}\right) & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix},\\
h_{1,t} &= \mu_1 + \phi_1(h_{1,t-1} - \mu_1) + \eta_{1,t}, \quad \eta_{1,t} \sim N(0, \sigma_{\eta_1}^2),\\
v_t &\sim N(0, \lambda_t^{-1} R), \quad \lambda_{1t} \sim Ga\left(\frac{\nu}{2}, \frac{\nu}{2}\right),
\end{aligned}
$$

with $v_t = [v_{1,t}, v_{2,t}]'$ and R the correlation matrix. Note that the covariance matrix Σ_t of v_t is given by:

$$
\Sigma_t = diag\left(e^{\frac{h_{1t}}{2}}, \sigma_2\right) R \, diag\left(e^{\frac{h_{1t}}{2}}, \sigma_2\right) = \begin{pmatrix} e^{h_{1t}} & \rho\sigma_2 e^{\frac{h_{1t}}{2}} \\ \rho\sigma_2 e^{\frac{h_{1t}}{2}} & \sigma_2^2 \end{pmatrix},
$$

We generate 100 observations for y_t from the above model with values of $g_{1,1} = g_{1,2} = g_{2,2} = 0.6, \rho = 0.7, \mu_1 = -5, \phi_1 = 0.9, \sigma_2 = 0.1 \ e \ \sigma_{\eta_1} = 0.05$.

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

Note that, as was done in Chib and Ramamurthy (2014), $g_{1,2}$ could not be identified by the likelihood function. To illustrate this point, in Figure 1 we plot the likelihood of the state matrix parameters $g_{1,1}, g_{1,2}$ and $g_{2,2}$, each varying for a grid of values and keeping the rest of the parameters fixed. Note in the plot of $g_{1,2}$ that the likelihood function is symmetric around zero, with peaks around -0.6 and 0.6. This problem could be solved by specifying an informative prior for $g_{1,2}$, but since our goal in this example is to estimate the volatility parameters, we fix the value of $g_{1,2}$ at its true value of 0.6.

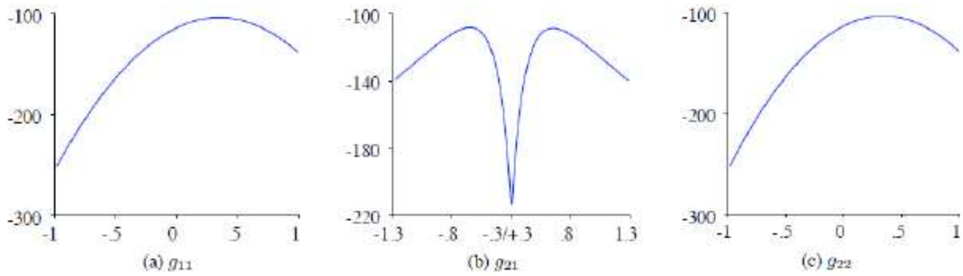

Figure 1: Log-likelihood for a grid of state matrix parameter values holding the other parameters fixed at their true values for a simple state-space DSGE model.

The prior distribution of $g_{1,1}$ and $g_{2,2}$ is given by a normal distribution with mean 0.1 and variance 2. For the volatility parameters we give the following priors: $\mu_1 \sim N(0,100), \rho \sim U[-1,1], \frac{\phi_1+1}{2} \sim Be(15,1.5), \sigma^2_{\eta_1} \sim IG(2.5,0.025)$, priors widely used in stochastic volatility models (Yu and Meyer, 2006). We will use the Metropolis-Hasting algorithm to estimate the parameters with 10,000 iterations; 500 warm-up and we take one in every 20 sampled values. Table I presents the posterior mean and 95% credibility interval.

Table I: Posterior mean and 95% credibility interval for the simple state-space DSGE model with Student's t-shocks and stochastic volatility.

	true value	*mean*	*CI 95%*
$g_{1,1}$	0.6	0.59	[0.37, 0.80]
$g_{2,2}$	0.6	0.6	[0.36, 0.81]
ρ	0.7	0.52	[0.21, 0.79]
σ_2	0.1	0.13	[0.09, 0.17]
μ_1	-5	-5.08	[-5.15, -4.91]
ϕ_1	0.9	0.82	[0.75, 0.91]
σ_{η_1}	0.05	0.05	[0.01, 0.08]
ν	5	12.79	[4.98, 32.82]

Note that, although there is little data generated, all parameters were estimated very well on average, with the exception of the degrees of freedom. For the purposes of model comparison, these same data were considered in the estimation of a normal model with stochastic volatility, which, in our simulated example above, corresponds to having $\lambda_t = 1, \forall t$. We will use the calculation of the logarithm of the marginal likelihood as a comparison criterion. Table II shows these values.

Table II: Marginal log-likelihood for the proposed models (Normal and Student-t with stochastic volatility) applied to the data generated from the simple DSGE model.

	Normal with volatility	*St-t with volatility*
Marginal log-likelihood	143.98	147.23

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

We calculate the Bayes factor as twice the difference of the logarithms of the marginal likelihoods. As stated in Kass and Raftery (1995), this quantity is on the same scale as the well-known likelihood ratio test statistic, which arises as an asymptotic approximation of applying Laplace's method, which yields an asymptotic chi-square distribution. Therefore, $2\ln(FB) = 2 * (147.23 - 143.98) = 6.5$. Following the criterion of Kass and Raftery (1995), this corresponds to strong evidence for the Student-t model over the normal model. Therefore, we can conclude that the stochastic volatility model can capture the nature of the data for a state-space model.

V. APPLICATION

Consider the model proposed by Ireland (2004) of a competitive equilibrium consisting of a representative household, a representative firm that produces a final product, a continuous number of intermediate firms and a central bank. We will give a simplified version of the model:

$$y_t = c_t + \frac{\phi}{2}\left(\frac{\pi_t}{\pi_s} - 1\right)^2 y_t, \quad (3)$$

$$\ln(a_t) = (1-\rho_a)\ln(a_s) + \rho_a \ln(a_{t-1}) + \epsilon_{at}, \quad (4)$$

$$\frac{a_t}{a_s} = \beta r_t E_t\left[\frac{a_{t+1}}{c_{t+1}}\frac{1}{z_{t+1}}\frac{1}{\pi_{t+1}}\right], \quad (5)$$

$$\ln(\theta_t) = (1-\rho_\theta)\ln(\theta_s) + \rho_\theta \ln(\theta_{t-1}) + \epsilon_{\theta t}, \quad (6)$$

$$\ln(z_t) = \ln(z_s) + \epsilon_{zt}, \quad (7)$$

$$0 = 1 - \theta_t + \theta_t\left(\frac{c_t}{a_t}\right)y_t^{\eta-1} - \phi\left(\frac{\pi_t}{\pi_s} - 1\right)\frac{\pi_t}{\pi_s} + \beta\phi E_t\left[\frac{a_{t+1}}{a_t}\frac{c_t}{c_{t+1}}\left(\frac{\pi_t}{\pi_s} - 1\right)\frac{\pi_{t+1}}{\pi_s}\frac{y_{t+1}}{y_t}\right], \quad (8)$$

$$g_t = \frac{y_t}{y_{t-1}} z_t, \quad (9)$$

$$x_t = \frac{y_t}{a_t^{\frac{1}{\eta}}}, \quad (10)$$

for all $t = 0,1,2,\ldots$ where $y_t = \frac{Y_t}{Z_t}, c_t = \frac{C_t}{Z_t}, \pi_t = \frac{P_t}{P_{t-1}}, z_t = \frac{Z_t}{Z_{t-1}}, g_t = \frac{Y_t}{Y_{t-1}}, x_t = \frac{Y_t}{Q_t}$. Where C_t represents household consumption of the final product purchased at a nominal price P_t from the representative firm, r_t denotes the nominal interest rate between t and $t+1$. a_t are shocks to household preferences. During each period t, the representative firm uses Y_t units of the intermediate good, θ_t is a random shock that sets the price above marginal cost. Z_t is an aggregate technological shock. Q_t reveals the efficient level of output, g_t is defined as the growth rate of output, and x_t as the output gap.

System (3)-(10) can be log-linearized around its steady state to describe how the economy responds to shocks. Let $\hat{y}_t = \ln\left(\frac{y_t}{y_s}\right)$, $\hat{c}_t = \ln\left(\frac{c_t}{c_s}\right), \hat{r}_t = \ln\left(\frac{r_t}{r_s}\right), \hat{\pi}_t = \ln\left(\frac{\pi_t}{\pi_s}\right), \hat{g}_t = \ln\left(\frac{g_t}{g_s}\right), \hat{x}_t = \ln\left(\frac{x_t}{x_s}\right)$, $\hat{a}_t = \ln\left(\frac{a_t}{a_s}\right), \hat{\theta}_t = \ln\left(\frac{\theta_t}{\theta_s}\right)$, $\hat{z}_t = \ln\left(\frac{z_t}{z_s}\right)$. A first-order Taylor approximation in (3) reveals that $\hat{c}_t = \hat{y}_t$, allowing $\hat{c}_t$ to be eliminated from the system. First-order approximation in the remaining seven equations leads to:

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_{at}, \quad (11)$$

$$\hat{x}_t = E_t\hat{x}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + \left(1 - \frac{1}{\eta}\right)(1-\rho_a)\hat{a}_t, \quad (12)$$

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \epsilon_{\theta t}, \quad (13)$$

$$\hat{z}_t = \epsilon_{zt}, \quad (14)$$

$$\phi\hat{\pi}_t = \beta\phi E_t\hat{\pi}_{t+1} + \eta(\theta_s - 1)\hat{x}_t - \hat{\theta}_t, \quad (15)$$

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t, \quad (16)$$

$$\hat{x}_t = \hat{y}_t - \left(\frac{1}{\eta}\right)\hat{a}_t, \quad (17)$$

for all $t = 0,1,2,\ldots$ The central bank conducts monetary policy by adjusting the short-term interest rate according to the rule:

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_t + \rho_g\hat{g}_t + \rho_x\hat{x}_t + \epsilon_{r,t.} \quad (18)$$

Following Ireland (2004), we can add a further lag in the output gap and inflation in equations (12) and (15) to obtain:

$$\hat{x}_t = \alpha_x\hat{x}_{t-1} + (1-\alpha_x)E_t\hat{x}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + \left(1 - \frac{1}{\eta}\right)(1-\rho_a)\hat{a}_t,$$

$$\phi\hat{\pi}_t = \beta\phi\alpha_\pi\hat{\pi}_{t-1} + \beta\phi(1-\alpha_\pi)E_t\hat{\pi}_{t+1} + \eta(\theta_s - 1)\hat{x}_t - \hat{\theta}_t,$$

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

Therefore, the model can be summarized in a system of eight equations:

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t,$$
$$\hat{\pi}_t = \beta\alpha_\pi\hat{\pi}_{t-1} + \beta(1-\alpha_\pi)E_t\hat{\pi}_{t+1} + \psi\hat{x}_t - \hat{e}_t,$$
$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_t + \rho_g\hat{g}_t + \rho_x\hat{x}_t + \epsilon_{r,t},$$
$$\hat{x}_t = \alpha_x\hat{x}_{t-1} + (1-\alpha_x)E_t\hat{x}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + (1-\omega)(1-\rho_a)\hat{a}_t,$$
$$\hat{y}_t = \hat{x}_t + \omega\hat{a}_t,$$
$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \epsilon_{at},$$
$$\hat{e}_t = \rho_e\hat{e}_{t-1} + \epsilon_{et},$$
$$\hat{z}_t = \epsilon_{zt}.$$

Note that ω is defined as $\omega = \frac{1}{\eta}, \psi = \frac{\eta(\theta_s-1)}{\phi}$ and the new shock $\hat{e}_t$ is defined as $\hat{e}_t = \left(\frac{1}{\phi}\right)\hat{\theta}_t$. In this system $\hat{g}_t, \hat{\pi}_t, \hat{r}_t, \hat{x}_t$ and $\hat{y}_t$ denote the growth of output, inflation, nominal interest rate, output gap and stochastic detrended output, respectively, and $\hat{a}_t, \hat{e}_t$ and $\hat{z}_t$ capture exogenous changes in preferences, production costs and technology, respectively. These equations represent the growth rate of output, the Phillips curve, the Taylor rule, the forward-looking IS curve, the growth rate of the output gap, the evolution of shocks to preferences, production costs and technology, respectively.

We denote the shocks by $\epsilon_t = [\epsilon_{a,t}, \epsilon_{e,t}, \epsilon_{z,t}, \epsilon_{r,t}]'$. The parameters z_s and π_s in the nonlinear model determine the steady state of output growth and inflation, respectively. Furthermore, β determines the steady state of the short-term nominal interest rate through the relation $r_s = \frac{z_s\pi_s}{\beta}$. The values of z_s, π_s and β will be fixed at the mean value of output growth, inflation and interest rate from the data. Also, ψ and ρ_r are fixed at 0.10 and 1.00, respectively. The basic model parameters of interest are:

$$\theta_1 = (\omega, \alpha_x, \alpha_\pi, \rho_\pi, \rho_g, \rho_x, \rho_a, \rho_e, \sigma_a, \sigma_e, \sigma_z, \sigma_r)'.$$

The data to be used are the series of log-deviations for output growth $\hat{g}_t$, inflation $\hat{\pi}_t$, and short-term nominal interest rate $\hat{r}_t$ with respect to the steady state over the period 1980:I to 2003:I for the U.S. economy. In Figure 2 we show this data.

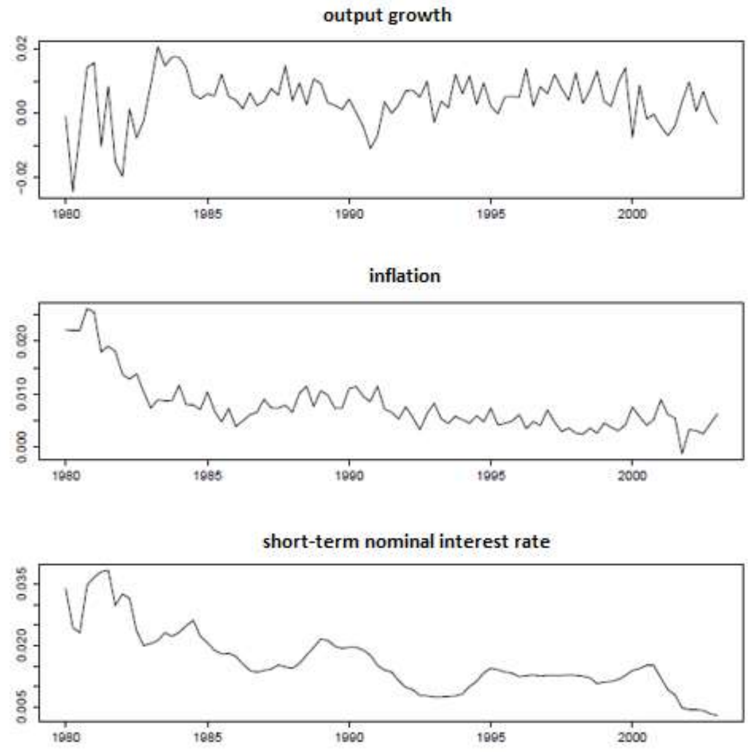

Figure 2: Quarterly data on U.S. output growth, inflation, and short-term nominal interest rates from 1980:I to 2003:I.

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

After applying Klein's Method to solve the system of rational expectations, the system can be written as a state space, where the states S_t are defined as:

$$S_t = [\hat{y}_{t-1}, \hat{r}_{t-1}, \hat{\pi}_{t-1}, \hat{g}_{t-1}, \hat{x}_{t-1}, \hat{a}_t, \hat{e}_t, \hat{z}_t, v_{rt}]'$$

The shock vector $v_t = \epsilon_t$ and the observation vector Y_t:

$$Y_t = [\hat{g}_t, \hat{\pi}_t, \hat{r}_t]'.$$

Therefore, the complete model is written as:

$$Y_t = HS_t,$$
$$S_{t+1} = GS_t + Fv_{t+1},$$
$$h_{t+1} = \mu + \Phi(h_t - \mu) + \eta_t, \qquad t = 1, \dots, n-1,$$
$$h_1 \sim N_p(\mu, \Sigma_0),$$

Where $v_t \sim N(0, \lambda_t^{-1}\Sigma_t), \lambda_t \sim Ga\left(\frac{v}{2}, \frac{v}{2}\right), h_t = (h_{at}, h_{et})'$, with $\Sigma_t = V_t^{1/2} R V_t^{1/2}$, and $V_t^{1/2} = diag\left(\exp\left(\frac{h_{at}}{2}\right), \exp\left(\frac{h_{et}}{2}\right), \sigma_z, \sigma_r\right), \eta_t | h_t \sim N(0, \Sigma_{\eta\eta})$, Φ and $\Sigma_{\eta\eta}$ diagonal matrices of 2×2 and *H, G* and *F* are matrices that depend on the structural parameters of the model. Note that, for this application, we are considering that only the first two components of the shock vector have stochastic volatility, that is, due to our experience in the simple DSGE example of the previous section, where the volatility model was well estimated only when only one component was considered. Thus, in this application it seems reasonable to assume volatility only in shocks that follow an autoregressive structure. In Cruz-Torres and Migon (2025) is made it this application for non structure variance.

A. Specification of Prior Distributions

Once the system is written as a state space, the Kalman filter can be applied to compute the likelihood function. To complete the inference, a prior distribution for the parameters must be specified to obtain the posterior distribution.

As pointed out in the literature, for DSGE models an informative prior is important because it adds mass in important regions of the parameter space, places where the likelihood function often cannot capture. We chose the same prior distributions that were used by Chib and Ramamurthy (2014), since we need to have a point of comparison to evaluate our model. Furthermore, they turn out to be very adequate priors for the parameters of this model. In Table III we specify the priors of the parameters.

Table III: Prior distributions of structural parameters in the model

Parameter	*distribution*	*mean*	*Standard deviation*
ω	Beta	0.20	0.10
α_x	Beta	0.10	0.05
α_π	Beta	0.10	0.05
ρ_π	Gamma	0.30	0.10
ρ_g	Gamma	0.30	0.10
ρ_x	Gamma	0.25	0.0625
ρ_a	Beta	0.85	0.10
ρ_e	Beta	0.85	0.10
σ_a	Inverse Gamma	0.055	0.055
σ_e	Inverse Gamma	0.0027	0.0098
σ_z	Inverse Gamma	0.022	0.037
σ_r	Inverse Gamma	0.0072	0.0144

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

We also need to specify the prior distributions for the volatility parameters. In Table IV we specify these priors, chosen following the arguments presented in Yu and Meyer (2006), which are priors widely used in stochastic volatility models. Finally, for the degrees of freedom ν, we use an independent prior such as the one proposed in Fonseca et al. (2008).

Table IV: Priors used for the parameters belonging to the stochastic volatility model.

Parameter	*distribution*	*mean*	*Standard deviation*
R	Uniform	0	0.577
μ_i	Normal	0	100
$(\phi_i + 1)/2$	Beta	0.91	0.069
$\sigma^2_{\eta,ii}$	Inverse Gamma	0.017	0.023

Therefore, our parameter set θ_2 is composed of the volatility and correlation parameters which are given by:

$$\theta_2 = \left(\mu_a, \mu_e, \phi_a, \phi_e, \sigma^2_{\eta,a}, \sigma^2_{\eta,e}, \nu, r_{ae}, r_{az}, r_{ar}, r_{ez}, r_{er}, r_{zr}\right)'$$

Finally, the set of parameters to be estimated θ, is formed by $\theta = (\theta_1, \theta_2)'$.

B. Results

We use the Metropolis-Hasting algorithm with random walk to simulate the posterior distribution of the parameters with an acceptance rate between 20-40% for all parameters. We will estimate four different models: diagonal normal model with stochastic volatility (M1), diagonal Student-t model with stochastic volatility (M2), normal model with correlation and stochastic volatility (M3) and Student-t model with correlation and stochastic volatility (M4). We run 100,000 iterations of the MCMC scheme. We discard the first 20,000 iterations and take one every 50 sampled values, such that we obtain a sample size of 1600 from the posterior distribution. Table V shows the point estimates and credibility intervals for the normal and Student-t models with correlation and stochastic volatility (M3 and M4). Note that, for the parameter set θ_1, the estimates are very similar between both models. Similarly, the parameter set θ_2 shows similar estimates. It is worth noting that the degree of freedom was estimated around 3, reinforcing the conclusion that fixing the degrees of freedom may not be a good practice.

Table V: Posterior mean and 95% credibility intervals estimated with normal model (M3) and Student-t model (M4), both assuming stochastic volatility.

	Model 3		*Model 4*	
Parameter	*mean*	*CI 95%*	*mean*	*CI 95%*
ω	0.0100	[0.001, 0.021]	0.0101	[0.000, 0.025]
α_x	0.0506	[0.002, 0.167]	0.0566	[0.003, 0.186]
α_π	0.0778	[0.006, 0.226]	0.0693	[0.001, 0.235]
ρ_π	0.3066	[0.061, 0.773]	0.2740	[0.048, 0.701]
ρ_g	0.4311	[0.188, 0.832]	0.5174	[0.218, 1.032]
ρ_x	0.1636	[0.041, 0.399]	0.1749	[0.043, 0.408]
ρ_a	0.9969	[0.994, 0.998]	0.9972	[0.994, 0.998]
ρ_e	0.8555	[0.658, 0.989]	0.8875	[0.706, 0.995]
σ_z	0.0126	[0.003, 0.037]	0.0125	[0.003, 0.036]
σ_r	0.0039	[0.001, 0.011]	0.0040	[0.001, 0.012]

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

μ_a	-1.5174	[-16.12, 13.46]	-1.5441	[-15.33, 12.37]
μ_e	-12.470	[-13.24, -10.32]	-13.061	[-13.75, -12.27]
ϕ_a	0.9728	[0.868, 0.999]	0.9579	[0.691, 0.999]
ϕ_e	0.7677	[0.617, 0.853]	0.7545	[0.596, 0.857]
$\sigma^2_{\eta,a}$	0.0125	[0.003, 0.040]	0.0137	[0.003, 0.044]
$\sigma^2_{\eta,e}$	0.1343	[0.016, 0.576]	0.1206	[0.019, 0.364]
ν	-	-	3.1025	[1.204, 7.082]
r_{ae}	0.1485	[-0.511, 0.683]	0.0447	[-0.602, 0.644]
r_{az}	-0.4292	[-0.863, 0.296]	-0.3697	[-0.892, 0.492]
r_{ar}	0.1270	[-0.538, 0.723]	-0.0587	[-0.673, 0.666]
r_{ez}	0.0283	[-0.778, 0.759]	0.2213	[-0.522, 0.856]
r_{ez}	0.0584	[-0.520, 0.582]	0.0719	[-0.437, 0.599]
r_{zr}	-0.2394	[-0.926, 0.798]	-0.0204	[-0.777, 0.714]

In Figure 3 we show the priors and posteriors for the parameters of the Student-t model with stochastic volatility (M4) (the priors are dotted lines, and the posteriors are solid lines). In Figure 4 we show the histograms of the posteriors of the correlations and the credibility intervals for the Student-t model with stochastic volatility (M4). Since a uniform prior was specified for the correlation matrix R, this does not correspond to a uniform marginal prior for each element due to the restriction of the correlation being positive definite.

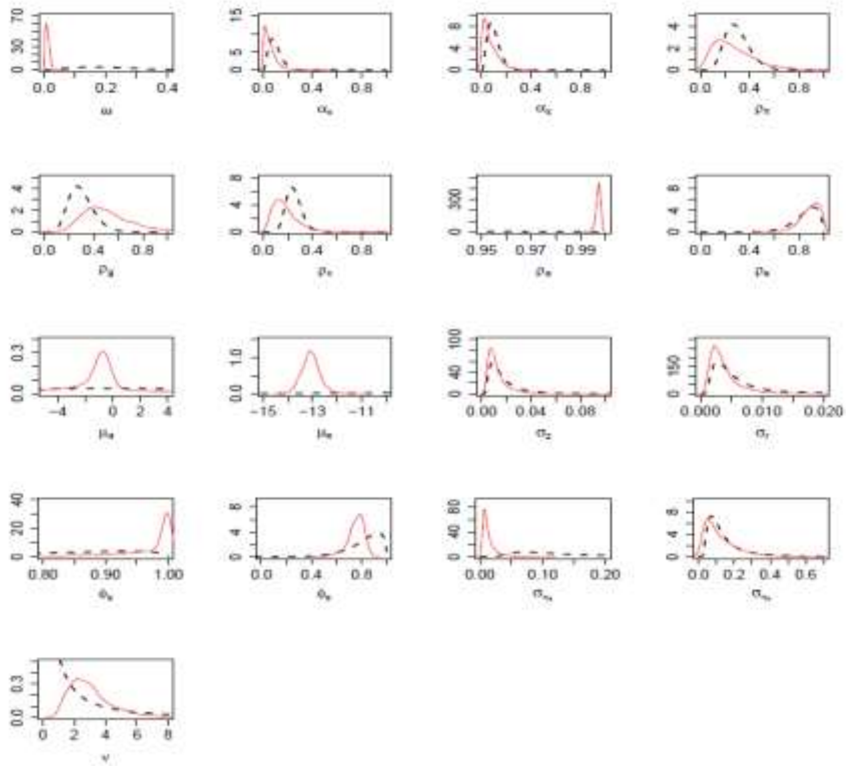

Figure 3: Prior (dotted line) and posterior (solid line) distribution for the structural parameters estimated with the Student-t model with stochastic volatility (M4).

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

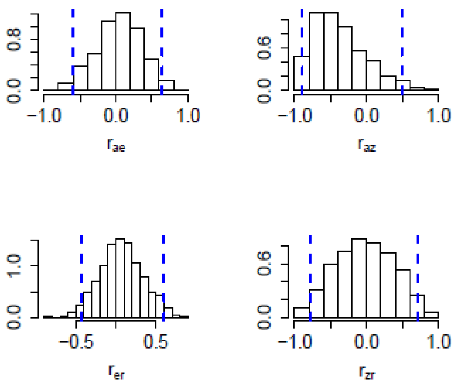

Figure 4: Histogram of the posterior distributions of the correlations estimated with the Student-t model with stochastic volatility (M4).

In Figure 5, we show the evolution of volatility for the two shocks considered: the preference shock and the production cost shock. Note that the preference shock varies over the years, which provides clear evidence that the variance varies over time. On the other hand, the production cost shock shows less variation in volatility over time, which could suggest that the variance is constant, especially since 1984, with the exception of a small peak in the early 2000s. Note that the estimated value of μ_e was -12.4 in the normal case (M3) and -13.0 in the t-Student case (M4), which corresponds to a standard deviation σ_e of 0.0020 in the normal case (M3) and 0.0015 in the t-Student case (M4). Therefore, if the production cost shock is in fact constant, the evolution of the variance already suggests this.

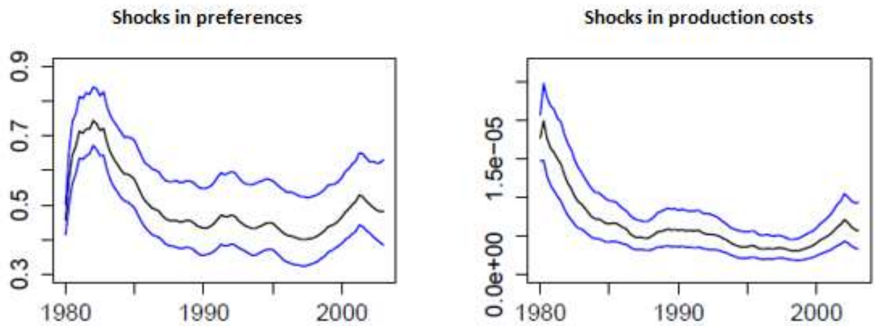

Figure 5: Estimated stochastic volatility and 95% credibility interval for preference and production cost shocks from the DSGE model.

In Figure 6, the posterior mean is shown, as well as its respective 95% credibility intervals of the mixing variable $\lambda_t, t = 1, \ldots, 93$ of the Student-t model with stochastic volatility (M4). Since in priori $\lambda_t \sim Ga\left(\frac{\nu}{2}, \frac{\nu}{2}\right), t = 1, \ldots, 93$ has an expected value $E(\lambda_t) = 1$, finding a posterior mean different from 1, $E(\lambda_{jt} | \ldots) \neq 1$, possibly implies the presence of a structural break in the economy. Since we have only one mixing variable, the presence of a structural break will not tell us which shock caused it. In Figure 6 we show the posterior means of the mixing variable. Let us note that there is some evidence of structural change in the early 1980s and early 2000s. Therefore, looking at the data in Figure 2, the large variability in the early 1980s and early 2000s in the product growth and inflation series may be caused, possibly, by some shock.

International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 08 Issue 09 September 2025
DOI: 10.47191/ijcsrr/V8-i9-17, Impact Factor: 8.048
IJCSRR @ 2025
www.ijcsrr.org

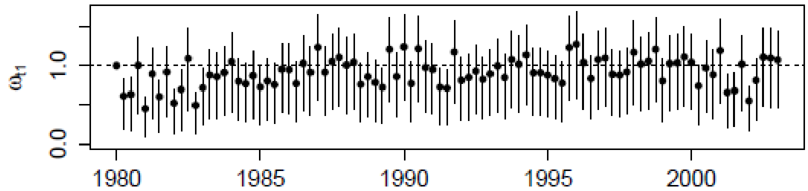

Figure 6: Posterior means and 95% credibility intervals of the mixture variables λ_{jt} of the Student-t model with stochastic volatility.

To compare the models, we use the logarithm of the marginal likelihood as a criterion. Table VI shows these values for the four models used.

Table VI: Marginal log-likelihood for the four proposed models: Diagonal normal model (M1), Diagonal Student-t model (M2), Correlated normal model (M3) and Correlated Student-t model (M4), all with stochastic volatility.

Model	*M1*	*M2*	*M3*	*M4*
Marginal log-likelihood	1227.3	1288.2	1316.0	1354.5

Note that these models with volatility and correlation are superior to those traditional normal models with constant variance, they are superior to those models with independent stochastic volatility.

VI. CONCLUSIONS

Throughout this article, we have detailed how a DSGE model can incorporate a structure into the variance. The stochastic volatility model is considered, since having a shock in the variance evolution results in greater freedom in the variance behaviour.

A simple economic example was tested, and it was noted that stochastic volatility models do not estimate well with few data. Therefore, considering a volatility structure in all shocks of the model is not ideal, but as an alternative one can consider volatility structure partially only in some of the shocks. An application of real data was made in four cases: shocks with independent normal distribution and stochastic volatility structure, shocks with independent Student-t distribution and stochastic volatility structure, correlated shocks with normal distribution and stochastic volatility structure, correlated shocks with Student-t distribution and stochastic volatility structure. We noted that the fit was better than considering constant variance over time.

REFERENCES

1. Asai, M., McAleer, M. and Yu, J. 2006. Multivariate stochastic volatility: A review. Econometric Reviews, 25, 145-175.
2. Barnard, J., McCulloch, R. and Meng, X. 2000. Modeling covariance matrices in terms of standard deviations and correlations, with applications to shrinkage. Statistica Sinica, 10, 1281-1311.
3. Blanchard, O. and Kahn, C. 1980. The solution of linear difference models under rational expectations. Econometrica, 48, 1305-1311.
4. Chib, S., Omori, Y. and Asai, M. 2009. Multivariate stochastic volatility. Handbook of Financial Time Series, 365-400.
5. Chib, S. and Ramamurthy, S. 2014. DSGE models with student-t errors. Econometrics Reviews, 33, 152-171.
6. Cruz-Torres, C. and Migon, H. 2025. DSGE model with non structure variance. Brazilian Journal of Development, 11(2), 1-21.
7. Curdia, V., Del Negro, M. and Greenwald, D. 2014. Rare shocks, great recessions. Journal of Applied Econometrics, 29, 1031-1052.
8. Fernandez-Villaverde, J., Guerron-Quintana, P. and Rubio-Ramirez, F. 2015. Estimating dynamic equilibrium models with stochastic volatility. Journal of Econometrics, 185, 216-229.

9. Fonseca, T., Ferreira, M. and Migon, H. 2008. Objective bayesian analysis for the Student-t regression model. Biometrika, 95, 325-333.
10. Ireland, P. 2004. Technology shocks in the new keynesian model. The Review of Economics and Statistics, 86, 923-936.
11. Jacquier, E., Polson, N. and Rossi, P. 2004. Bayesian analysis of stochastic volatility models with fat-tails and correlated errors. Journal of Econometrics, 122, 185-212.
12. Justiniano, A. and Primiceri, G. 2008. The time-varying volatility of macroeconomic fluctuations. American Economic Review, 98, 604-641.
13. Kass, R. and Raftery, A. 1995. Bayes factor. Journal of the American statistical association, 90, 773-795.
14. Klein, P. 2000. Using the generalized Schur form to solve a multivariate linear rational expectations model. Journal of Economic Dynamics and Control, 24, 1405-1423.
15. Uhlig, H. 1999. A toolkit for analyzing nonlinear dynamic stochastic models easily. Computational Methods for the Study of Dynamic Economies.
16. Yu, J. and Meyer, R. 2006. Multivariate stochastic volatility models: Bayesian estimation and model comparison. Econometric Reviews, 25, 361-384.

Cite this Article: Cruz Torres, C.A. (2025). DSGE Model with Structure Variance. International Journal of Current Science Research and Review, 8(9), pp. 4544-4556. DOI: https://doi.org/10.47191/ijcsrr/V8-i9-17