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A Theoretical and Practical Analysis of Geometry and Proportions in Discontinuous Double-Layer Architectural Systems

Mohammad Maruf Sharifi[1*], Mohammad Sharif Omari[2]
[1]Geometry and technical drawing Department, Faculty of Construction, Jawzjan University, Afghanistan
[2]Architecture Department, Faculty of Construction, Jawzjan University, Afghanistan

ABSTRACT: Despite the stylistic and structural significance of discontinuous double-shell domes in architectural heritage, limited scholarly attention has been devoted to their geometric foundations. This case-based study investigates the relationship between theoretical and practical geometry in the design and construction of discontinuous double-shell domes. The research seeks to answer two main questions: (1) What is the connection between theoretical and applied geometry in the shaping of such domes? (2) What are the key factors influencing their design and construction process? The study begins with a review of the theoretical foundations of geometry in architecture, followed by an analysis of a selected case study to trace the application of geometric and proportional principles derived from the theoretical phase. Through this analysis, the underlying geometric logic of the dome's formation is decoded. The results reveal that the dome was designed based on a premeditated geometric system involving fundamental shapes—square, circle, and pentagon—with proportions governed by the golden ratio. These proportions are consistently manifested in the plan, section, and elevation. The study contributes to the understanding of traditional geometric practices and their potential application in the conservation of historical domed structures as well as the design of contemporary shell architecture.

KEYWORDS: Architectural Geometry, Double-Shell Dome, Geometric Design, Indigenous Knowledge, Practical Geometry, Proportions, Structural Analysis, Traditional Architecture ,Theoretical Geometry.

INTRODUCTION

Since the dawn of humanity, awareness of the environment and the quest to create harmony within it have led to the exploration of proportions as a fundamental concept in art and architecture. Proportions represent a mathematical relationship between elements of a work, contributing to visual order, balance, and aesthetic appeal. Historically, architects have employed mathematics and geometry to establish such proportions, enabling them to design structures that exhibit coherence, hierarchy, and elegance across various architectural applications.

Domed structures, as one of the most significant architectural elements, embody specific geometric and arithmetic proportions that skilled architects have developed and refined throughout different historical periods. These buildings have continually attracted scholarly attention for their aesthetic and structural qualities. Analyzing the geometric proportions within domed architecture allows for the identification of design patterns and hierarchies, facilitating a deeper understanding of both historical construction processes and modern architectural applications.

This study investigates the relationship between geometric proportions and the design of double-shell domes, addressing key questions: What is the connection between geometric proportions and dome design? What are the main factors influencing the design and construction processes? Are there shared proportional patterns among various domed structures? By examining the dimensions and hierarchical relationships between architectural elements, this research aims to decode the underlying geometric logic that governs dome construction.

Geometry, understood as the science of measuring and spatial relationships, has been an implicit and explicit framework guiding architectural practices both theoretically and practically. Among architectural forms, the dome stands out for its symbolic prominence, structural complexity, and visual distinctiveness. Its geometric composition, influenced by concepts such as height and span, significantly impacts the proportional system and overall design.

In this context, the present research aligns with the theoretical and practical analysis of geometry and proportions in discontinuous double-shell domes, aiming to contribute to the preservation of traditional architectural knowledge and the advancement of modern shell structures.

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MATERIALS AND METHODS

In this study, an experimental and library-based research method was employed to test the conformity of the dimensions and measurements of architectural elements with geometric proportions. Initially, to establish the theoretical foundations of the research, the design systems and construction technologies of dome-shaped buildings were examined and analyzed through previous sources and studies. Then, architectural plans and sections were utilized for the geometric analysis of the design in both plan and cross-section views.

Furthermore, the geometric order and proportional relationships governing the buildings were analyzed. This geometric order was identified and presented based on a logical method through the analysis and comparison of drawings in plan, section, and elevation. Ultimately, the research objectives were pursued and fulfilled.

The presentation of architectural drawings and visual documentation derived from field studies and surveys plays a decisive role in this research by enabling comparison and inference of results.

Geometry and Proportions

Geometry is a suitable tool for organizing architecture and establishing conscious relationships among the components of a building.

When examining the concept of the term "geometry," numerous definitions emerge, all of which emphasize the use of measurement in some form. Geometry is the Arabicized form of the word "measurement" and refers to the knowledge that deals with the characteristics and relationships of shapes and sizes (Noghrehkar, 2013, p. 209).

The word "geometry" originates from the root meaning "measurement" and in the Pahlavi language was called "Hendāchak" (Farhoushi, 1973, p. 75). In European languages, the term has Greek origins and comes from "geometria," meaning the measurement of the earth (Hejazi, 2009, p. 15). Al-Khwarizmi considers geometry to be the Arabicized form of the word "measurement" (Arian, 2005, p. 93). Al-Munjid attributes a Persian root to this word (Abul-Qasemi, 1987, p. 364).

The original root of this word is "Zamig Paymana," where the first word means "earth" and the second means "measurement" (Mackenzie, 2009, p. 169). In dictionaries, the term is also defined as shape and size (Moein, 1981, p. 3258; Khalaf Tabrizi, 1982, p. 702; Dehkhoda, 1998, p. 23559).

Ibn Sina (Avicenna) defines geometry as the science of recognizing the position of lines, shapes, surfaces, and proportions (Ibn Sina, 2027, p. 88). Like other sciences, geometry originates from observation and experience and has a strong connection to human economic needs (Molavi, 2002, p. 12). Observation here refers to careful scrutiny of the surrounding world to learn from nature in order to meet human needs.

Al-Biruni defines geometry as the knowledge of measurements and properties of figures and shapes existing within physical bodies (Parsa, 2013, p. 2).

Proportions mean the appropriate relationship among parts with each other and with the whole work. Geometry and proportions are inseparable parts of architecture because creating movement within a regular geometric system is essential. None of the above definitions imply numerical application of measurement; rather, measurement means a ratio of a base amount, which can be a natural index or a part of the human body.

Accordingly, proportion in past measurement systems can be defined as the production of a new measurement by creating relationships between previous measurements.

Theoretical and Practical Geometry.

Geometry has always had two distinct aspects and has developed in two fields: the theoretical and mathematical aspect (theoretical geometry) and the practical and empirical aspect (practical geometry). Theoretical geometry is the form and shape that takes place in the architect's mind. In contrast, practical geometry refers to construction technology and a set of techniques and rules that assist designers in creating and organizing the design and make the construction and implementation process possible.

The importance of understanding the theoretical geometry of a building is such that it can be said that until the geometric proportions in the plan and elevation of the building are deciphered, one cannot gain a proper understanding or description of the building. On the other hand, studies show that practical geometry influences the design system of traditional architecture to such an extent that recognizing it is helpful for better understanding the lost parts of the building (Wolibeg et al., 2018, p. 1).

Architects in practical geometry have always used simple methods and tools to implement theoretical geometry. The only tools available to predecessors were the star and the compass, but architects commonly used string instead of the compass (Pirnia, 1993, p. 43).

The Role of Geometry in Roofing Design

Throughout all stages of the formation of an architectural work, a close relationship is observed between geometry and composition. The highest duty of the architect is to recognize, understand, and spatially visualize the static and dynamic forces within the load-bearing structure of the building (technical knowledge), and with full mastery, accurately determine the proportions and dimensions of the filled and empty spaces (Abulqasemi, 2006, p. 366).

This type of application of geometry in the design of coverings and the proportions and dimensions of the filled and empty parts represents the technical use of geometry in architectural design. The result of such a process is a type of architecture that is comprehensible in terms of order and proportions (Dehar & Alipour, 2013, p. 34).

The building's covering is the most premeditated and influential factor in shaping the building (Mehrian, 2018, p. 28). From a compositional and technical perspective, the design and execution of the covering influence the entire building and are the fundamental factors shaping the load-bearing elements and architectural space.

One of the most influential factors in the design process of the discontinuous double-shell domes of Nar, due to their dimensions and other characteristics, is the geometric factor. In the composition of the building, dimensions are derived from experiences gained over centuries, made possible by the cooperation of geometry and proportions (Abulqasemi, 2006, p. 369).

For example, for the interior domes, the basis of decision-making and action has been the span size, and based on the span size, the type of "chafad" (a structural element) was determined.

Analysis of Theoretical and Practical Geometry of the Dome and Domed Chamber

- **Determining the Overall Dimensions of the Dome Chamber and the Iwan in the Plan**

Analyzing geometric relationships in significant architectural structures reveals the architect's method of thinking and decision-making in addressing problems and finding appropriate solutions. It also demonstrates the ability of geometric reasoning to organize the architectural design process (Dehar & Alipour, 2013, p. 34).

Geometric analysis is a fundamental aspect of architectural analysis. Unless we accept that historical designers used specific geometric constructions in their design process, referring to geometry for analyzing such works cannot be justified (Pourahmadi, 2010, p. 85).

An architect's mastery of geometry and its creative use is essential for translating ideas and concepts into space and form. It defines the structure of the design and construction process, as geometry is a tool for shaping ideas into tangible reality.

Geometric analysis of any structure involves two main stages:

1. **The first stage** is identifying the key points of the design.
   Studies of architectural geometry reveal that architects traditionally began the design process by determining the geometric placement of key points within the overall plan. These points hold greater importance in the design, and additional spaces and elements are developed in proportion and harmony with them.
2. **The second stage** involves identifying the basic shapes and proportions used to define the dimensions of spaces and elements.
   Architects typically began their designs with a set of basic shapes, and spatial development was achieved through repetition, rotation, resizing, and combining these shapes.

The basic geometric shapes most commonly used in architecture include the **square**, rectangle, and circle in the plan, and the pentagon, hexagon, and **octagon** in sections and elevations.

In the case of the Abbasi Jame Mosque, one of the foundational points is the **center of the main** dome. Since the dome chamber is only one part of the mosque's total space, understanding how it was positioned within the entire plan required an initial examination of its geometric placement in the master plan. Subsequently, the geometric proportions of this part were analyzed and decoded.

In general, the square, circle, and **pentagon** are identified as the guiding forms in the design of the dome chamber. Proportional relationships between elements were established using the **special** golden rectangle, the 2:1 rectangle, and the golden ratio (1.618), which are explained in further detail.

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To locate the center of the dome within the general square-shaped plan (Figure 3), arcs were drawn from points C and D (midpoints of the square's sides) toward points A and B, with radii equal to AD and BC. These arcs intersect at point E, which marks the center of the dome in the plan. The extension of these arcs along side AB determines the width of the mihrab.

In the overall layout, the 2:1 ratio is observed in many spaces. To determine the dimensions of the dome chamber and the iwan (rectangle ABCD in Figure 4), the building's width was divided into four nearly equal parts. The middle half was designated for the dome chamber and surrounding halls (shabestans). This middle half was again divided into four equal parts, and the dome chamber and iwan were assigned to its central half.

As shown in Figure 5, the section of the dome chamber and the iwan forms a square with dimensions equal to the height of the dome. Therefore, the length of the dome chamber and iwan (AD and BC in Figure 4) was considered equal to the dome's height.

Figure 1. 3D Visualization

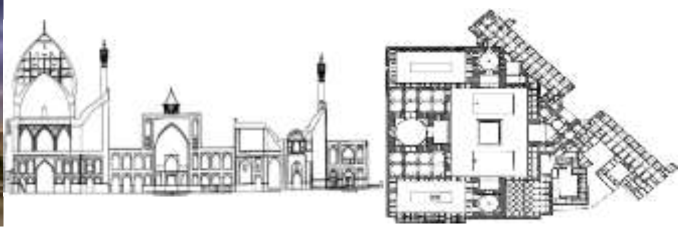

Figure 2. Architectural Plan and Section Drawing

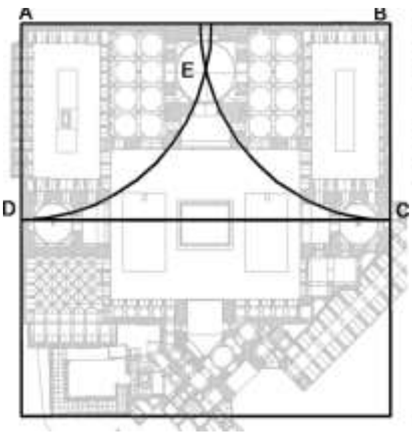

Figure 3. Determining the Center of the Dome in the Plan

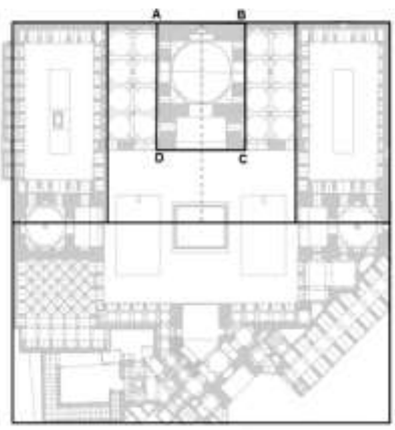

Figure 4. Determining the Dimensions of the Dome Chamber and the Iwan

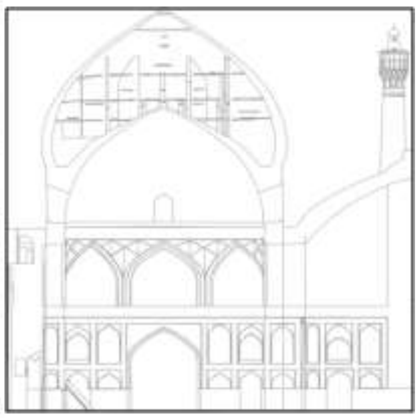

Figure 5. Dimensions of the Section of the Dome Chamber and the Iwan Space

❖ Geometric Analysis of the Dome Chamber Plan

In architecture, the design process is fundamentally based not on arithmetic calculations but on a set of geometric drawings. Such design requires a unified measurement system employed by the architect as a generating unit. All critical dimensions, both horizontal and vertical, relied on and were measurable by this unit.

When designing a large dome chamber, the length of its side (the dome's diameter) is considered as the generating unit. In the studied building, to determine the dimensions of the dome chamber and the iwan, the locations of these spaces were first identified along the main axis of the structure. The considered dimensions for the dome chamber are 22.5 by 22.5 meters. The ratio between this space and the adjacent shabestans (halls) and the iwan is 1:3 in length and 1:2 in width. Therefore, the dome chamber's space generally maintains a 3:2 ratio with the southern roofed space, a common proportion in architecture (see Figure 6).

Geometric analyses of the dome chamber and iwan plans revealed that, to determine the positions and sizes of the openings, a rule known as the "right-angled root 5" (radical ratio of 5) or the "half-square rule" was used (Golombek & Wilber, 1995, p. 200).

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The iwan, with approximate dimensions of 12 by 18.6 meters, has proportions close to the golden ratio (18.6/12 ≈ 1.6). As shown in Figure 6, to locate point A, an arc is drawn from center O with radius OO', defining the geometric position of point A along line KK'. Similarly, to find point B, another arc is drawn from O to O' with radius OO', intersecting KK'. The length AB defines the width of the iwan's entrance to the dome chamber.

To determine the geometric position of point L', an arc is drawn from center K' with radius AK', intersecting line O'K' at L'. For point M', an arc from K' with radius AK'/2 intersects O'K' at M'. Similarly, point N' is located by drawing an arc from K' with radius O''K', intersecting O'K' at N'. These constructions are symmetrically mirrored on the other side to complete the iwan's geometry in the plan.

Next, based on the iwan's proportions, the geometric proportions of the dome chamber's plan are determined. The quadrilateral AA'B'B has a 2:1 ratio. To locate points A' and B', segments AA' and BB' are drawn to half the length of AB. To find point A'1, an arc is drawn from O1 with radius O1A', intersecting segment O1O4 at A'1. Similarly, to determine point B'1, an arc is drawn from O2 with radius O2B', intersecting segment B'B'1.

In the dome chamber, a perfectly symmetrical space, the lengths O1A' or O2B' are considered as the base length, and drawings are constructed on all four peripheral sides accordingly. To find point P, an arc is drawn from O1 with radius (1/3) O1B'. Corresponding points to P on all four facades of the dome chamber are determined similarly.

Thus, it is observed that the dome chamber and iwan plan, despite the seemingly complex geometric details, are designed simply by establishing an overall 2:1 proportion and repeating a geometric rule and ratio.

❖ Geometric Analysis of the Section of the Dome Chamber and the Iwan

After the overall spatial division, the geometric analysis of the dome chamber space reveals that the implementation of subspaces also employs a special golden rectangle and the golden ratio (phi number). For example, the location of the dome center in the plan is positioned within the last third of the building based on space division by the golden ratio and the use of the phi number. To demonstrate this, the following steps are undertaken:

The rectangle ABCD in Figure (7) has sides with a 2:1 ratio. The diagonal AC of the rectangle is drawn, then an arc is drawn from center A with radius AD until it intersects diagonal AC at point O. From center C and radius OC, another arc is drawn that intersects side CD at point E, dividing this side according to the golden ratio. This point E defines the dome center on the plan. Furthermore, the extension of this point passes through the centers of the arches, the Greev window, and the dome (line EF), defining the geometric locus of these points.

Given that the applied order aligns perfectly with mathematical calculations, it is unlikely that the designer created this arrangement without awareness of these proportions. As observed, even for implementing practical geometry in hidden parts such as the space between the two shells in the initial design, the necessary foresight was applied.

For example, in rectangle ABCD in Figure (8), the arc OE divides side CD according to the golden ratio. If a perpendicular OF is drawn from point O, it intersects the shell (Ahyaneh) at point F, where the shell thickness reduces by one thin brick and also where the end of the large Kheshkhashi arch contacts the shell. If a perpendicular is drawn from point E, its extension intersects the shell at point G, the starting point of the Kheshkhashi arch and its intersection with the shell. If the segment EG is horizontally translated by the length of segment FG to the right, it defines the geometric locus of the outer shell of the Greev, tangent to it.

This location, where the shell thickness decreases and the shell thins, corresponds to the section divided by the golden ratio on the plan. Thus, the relationship between theoretical and practical geometry can be observed, proving the architect's need for theoretical geometric proficiency to implement practical geometry.

In rectangle ABCD in Figure (9), arc OE divides side BC at point E according to the golden ratio. Also, length CE defines the height of the Iwan's Chafad (structural element) from the floor.

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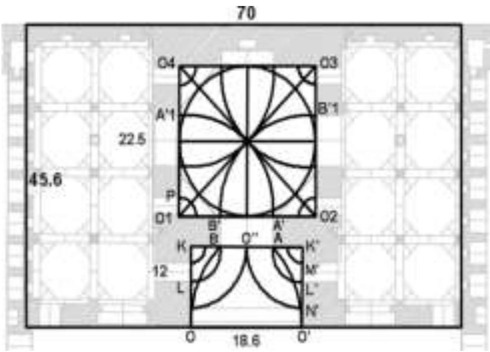

Figure 6. Geometric Analysis of the Dome Chamber and Iwan Plan

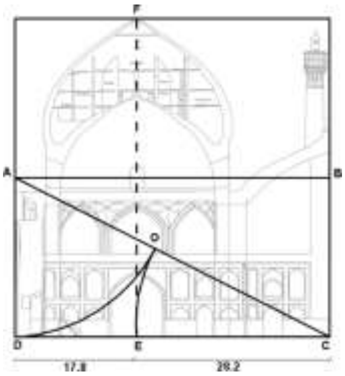

Figure 7. Geometric Location of the Dome Chamber Center

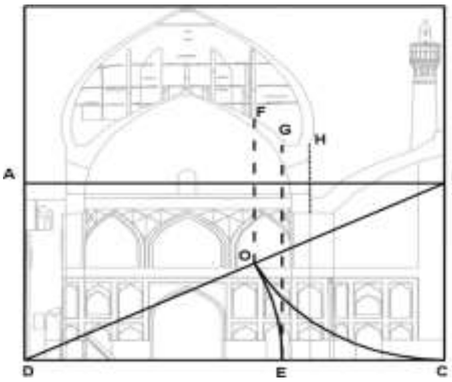

Figure 8. Geometric Analysis of the Section of the Dome Chamber

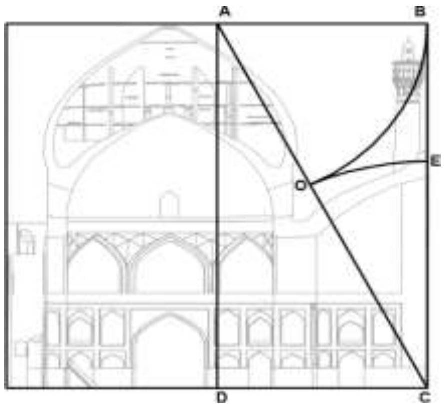

Figure 9. Proportions in the Iwan

C/DE=DC/EC=28.2/17.8= 46/28.2=1.62~1.618

Analysis of Theoretical and Practical Geometry of the Discontinuous Double-Shell Dome

- ❖ **Analysis of Proportions in the Shells**

Geometric Analysis of the Discontinuous Double-Shell Dome: Hidden Relationships and Proportions

In the study of architectural structures, geometry is often perceived through direct observation of the building's elevation, either explicitly or through the evocation of aesthetic sensation. However, the geometry present in the plan and concealed parts of the structure is not always easily perceived, as understanding the harmony of a building's geometry as a whole can be quite challenging. Some spatial relationships are not easily recognizable because they do not lie within a single visual field (Golombek & Wilber, 1995, p. 283) or are located in parts of the structure not directly visible—such as the internal space between the shells in discontinuous double-shell domes. In some cases, understanding the geometry of a structure requires awareness of the design process itself.

For the geometric analysis of a discontinuous double-shell dome—including the outer shell (*khod*), inner shell (*āhyāneh*), and the interstitial space—according to Figure 10, a circle is drawn with center *O* and radius *OA*. This circle defines the maximum diameter of the dome (outer shell). Inscribed and circumscribed pentagons are then constructed within and around the circle. It is observed that the dome's maximum height occurs at point *B*, which is exactly at the midpoint of line *B'B''*, connecting the top vertices of the two pentagons.

Connecting vertices *C'* and *D'* of the inscribed pentagon bisects line *A'B'* at point *O'* in the golden ratio (Relation 1). Point *O'* represents the geometric location of the *āhyāneh* apex and divides *A'B''* in the golden ratio (Relation 2). Thus, the height of the *khod* is derived from its maximum diameter, proving the use of the golden ratio in the dome's height and width.

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Similarly, connecting *C''* and *D''* of the circumscribed pentagon bisects *A''B''* at point *O''* in the golden ratio (Relation 3), which represents the apex of the small decorative finials (*khoshkhashi*). The height of *C''D''* corresponds to the beginning of the ornamental zone and the base of the *tonban* (a widening part of the minaret supporting the lantern), as noted by Pirnia (1991, p. 145).

Points *E'*, *A'*, and *F'* in the inscribed pentagon indicate the vertices of the arches bearing the dome. Line *G'G''*, the intersection of the pentagon diagonals, marks the top of the clerestory windows. Line *H'H''* specifies their width.

To determine the floor level of the dome chamber within the inter-shell space (Point *C*), diagonals *B'E'* and *C''F''* from the pentagons intersect at *D*. The horizontal extension *CD* yields this section's height.

To determine the height of the dome chamber's entry window in the inter-shell space (Point *E*), diagonals *B'E'* and *C'F'* intersect at *F*. The horizontal projection *EF* gives the height of the entry window.

To find the curvature of the *āvgun* (reverberation cavity), a vertical line is drawn from vertex *D'* of the inscribed pentagon to intersect the *khod* at *G*, and line *D'G* to the outer shell shows the curvature amount.

For maximum (*HI*) and minimum (*IJ*) thicknesses of the clerestory, extend line *D'G* downward. Then draw a vertical from *F''* to intersect side *D'F'* of the inscribed pentagon at *H*. Line *HI* marks the maximum thickness. A vertical from point *C* intersects *HI* at *J*, giving the minimum thickness *IJ*.

To determine the pier thickness supporting the dome, extend line *CG* to intersect side *D''F''* of the outer pentagon at *K*, then draw a vertical from *D''* downward, parallel to *JK*. Line *KL* (the horizontal connector) gives the pier thickness.

The *khod* and *āhyāneh* have variable thicknesses along their height—thickest at the base and thinning towards the apex. This gradual reduction likely required a series of control points. Studies show that where *khod* and *āhyāneh* intersect the diagonals of the pentagons, their thickness equals the vertical distance between the diagonals: *aa' = aa''* in the *āhyāneh*, and *bb' = bb''* in the *khod*.

These proportional relationships clearly demonstrate the architect's awareness of structural, constructional, and geometric principles in both theoretical and practical geometry during the design stage.

❖ Analysis of Proportions Between the Dimensions of Finials and the Dome Chamber Space

As previously explained, the building features three types of finials with different dimensions (see Figure A11). The conducted study on the heights of the finials and the interior height of the shell showed that the proportional relationship among the finial heights follows the golden ratio (phi) (Relations 3 and 4 in Figure 10). The ratio of the total height of the dome system (the shell and the dome) to the height of the large finials is 1.2, which is based on the golden ratio.

Additionally, point X4' not only represents the geometric location where the finials contact the shell but also indicates the part of the shell that has been thinned by the thickness of one brick.

The study of horizontal and vertical proportions between the dome chamber and the iwan on the roof of the mosque demonstrates relationships between the proportions of the dome chamber, the iwan, and the minarets.

To determine point K1, which is the lower edge of the iwan's pediment, a vertical line is drawn from point G perpendicular to the vertical diameter of the circle until it intersects at point K1'. If a line segment equal in length to GK1' is drawn from point G towards the right, point K1 is determined.

To find point K2, the end of the iwan arch, a horizontal line is drawn from point K2'', where the dome shell has its greatest curvature (this point's distance from G is twice the vertical distance CG) intersecting the vertical diameter of the circle at point K2'. Extending segment K2''K2' to the right by its own length locates point K2.

To determine point K3, starting from point K2', half the distance of K2'K1' is measured along the vertical diameter of the circle to find point K3'. Drawing a horizontal line from this point intersects the outer shell of the dome; extending this line by its own length locates point K3.

To obtain point K5, a horizontal line is drawn from the contact point of the circle and the circumscribed pentagon, point K5', to intersect the opposite outer shell at point K5''. Extending segment K5''K5' to the right by its own length gives point K5.

Points M and N represent the geometric intersection points of the shell and the circle, where the dome's shell thickness has been reduced by the thickness of one brick.

To find points K6 and K7, horizontal lines are drawn from points N and B' to intersect the axis of the minaret, thus locating these two points as well.

As observed, the design basis in the space between the two shells—which contains many hidden structural elements—is also grounded in geometry and proportions. These proportions are meticulously respected even in the smallest design details. All

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elements, both from part to whole and whole to part, maintain geometric proportions that, in an integrated system, result in a dome structure with a remarkably wide opening and impressive height and stability. This exemplifies the architect's mastery of theoretical geometry in implementing practical geometry once again.

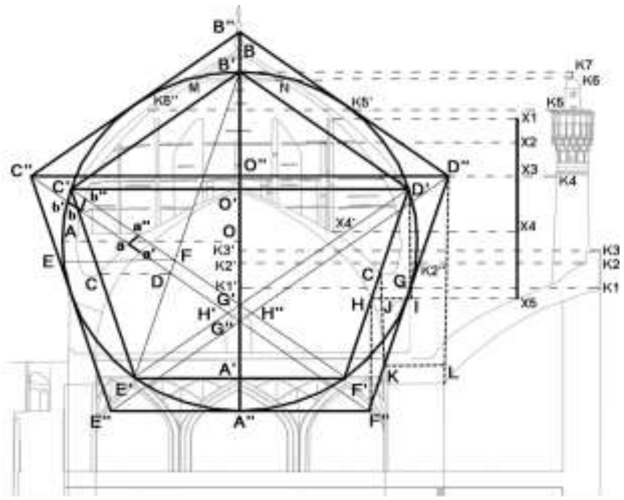

Figure 10. **Analysis of Theoretical and Practical Geometry of the Discontinuous Double-Shell Dome**

$A''O'/O'B=17.90/11=1.62\sim 1.618(1)$ $A'O'/O'B'=19.185/11.856=1.6181(2)$

$A''O''/O''B''=15.44/9.54=1.6182(3)$ $X1X3/X2X3=5.015/3.10=1.6177\sim 1.618\ (4)$

$X1X4/X4X5=9.168/5.66=1.618\ (5)$

Geometric Analysis of Finials

Placement of Finials in the Plan Utilizes the Basic Square Shape. As seen in (Figure 11B), the main finials align along the diagonals of two squares—one rotated and the other at a 45-degree angle. The secondary finials are located on the diagonals of the octagon formed by these two squares (Figure 11C). On each side of this octagon, two small finials are placed at nearly equal intervals, dividing the sides into three parts.

Only on the right side of the mihrab is the finial repetition pattern slightly broken. In this area, instead of two small finials between the large finial and the secondary finials, there is only one finial. To reduce the distance between the primary and secondary finials here, the primary finial has been shifted toward the secondary finials and does not sit at the square's vertex. Because of this, one vertex is effectively removed, transforming the 16-sided polygon inscribed in the octagon into a 15-sided polygon (Figure 11D).

Here, the geometric progression of (4), 8, (16) can be observed in the theoretical geometry design of the finials. Considering the principle of symmetry in the building and the necessity for balanced load distribution across various parts, it can be concluded that the initial plan for finial placement was based on polygons with 4, 8, and 16 sides. Had there been 15 small finials from the start, the design would have been different to maintain final symmetry.

This suggests that during construction, the architect altered the plan in this section for some reason. To prevent significant asymmetry and without changing the load distribution, a small finial was removed, and by slightly shifting the finials on the left half of the plan, the created gap was redistributed among the remaining finials. Removing the small finial lightens the structural load in that area, so it does not cause a problem. If instead of removal, an additional element had been added or element sizes increased, it could have caused point load increase, leading to stress concentration and structural disruption.

Geometry, as the science for choosing structural dimensions and components, governs the structural behavior of the building—behavior that follows geometric principles. A perfect geometry guarantees stability, which is a fundamental principle in traditional construction (Hejazi, 2009, p. 30).

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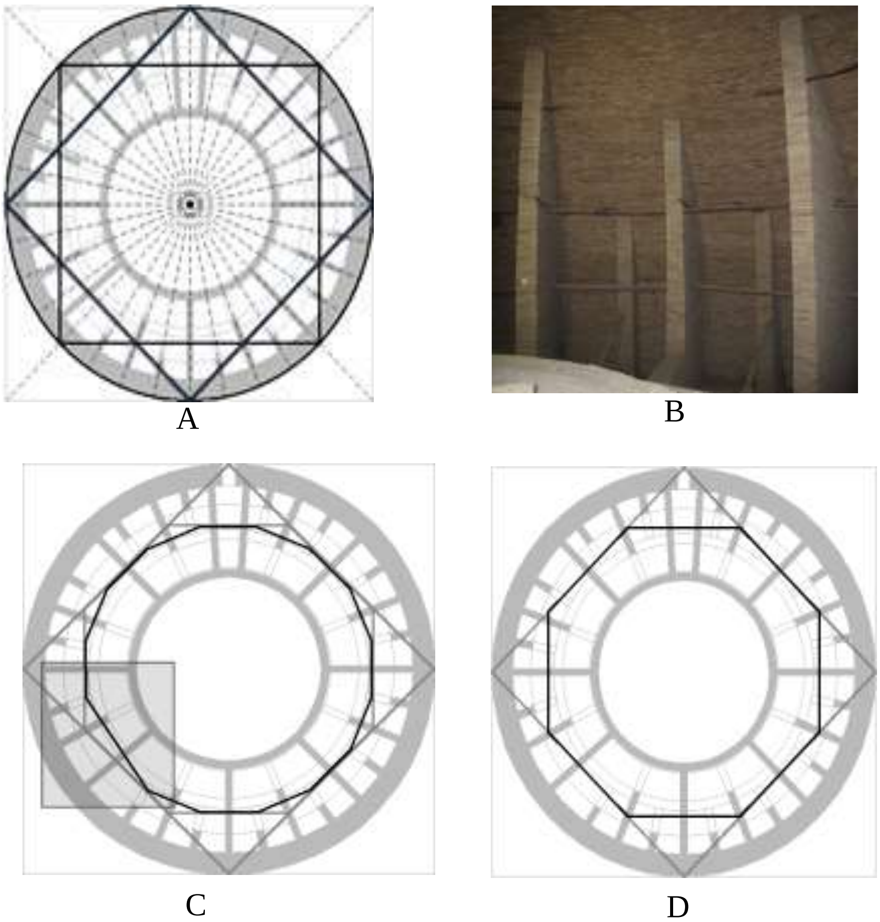

Figure 11. Placement and Proportions of Finials in the Plan

- **Pentagonal Section on the Plan of the Space Between the Two Shells**

  By matching the plan and the section of the space between the shells, the pentagon drawn in the section can also be projected onto the plan, and the corresponding points can be identified. Therefore, it can be concluded that the constructions and proportions present in the dome chamber have also been observed and applied in the plan, and vice versa. This, in turn, indicates the correlation between the geometric design in the plan and the elevation. The proportions in the plan and section are consistent and reinforce each other. Furthermore, this geometric harmony and alignment suggest that the design was conceived three-dimensionally from the beginning—meaning the architect visualized the overall volume of the building during the design process (see Figure 12).

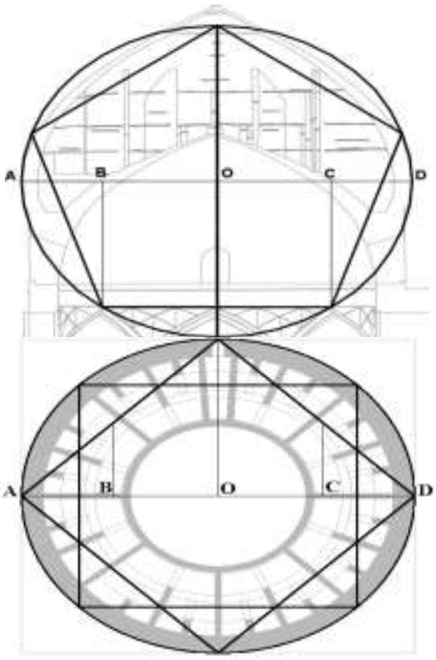

Figure 12. Superimposing the Sectional Pentagon onto the Plan of the Inter-Shell Space

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Geometric Analysis of the Elevation of the Dome Chamber

- **Finding the Main Dimensions in the Elevation**

Geometrical Analysis of the Elevation and Determination of Main Dimensions

To analyze the geometry of the elevation, it is first necessary to determine how the dimensions and spatial divisions of the elevation are derived from the plan. This is because the proportions in the plan define the proportions in the elevation. A study of the overall plan based on a top-down approach reveals that the width of the elevation has a 2:1 ratio with the total width of the building—a ratio that is consistently observed throughout the entire structure. Accordingly, the elevation, with a width of 70 meters and a height of 47 meters, is positioned in the central axis of the building. The width-to-height ratio of the elevation is 1.5, which is equivalent to a 3:2 proportion. Therefore, the final height of the dome has been determined based on these proportions (Rectangle ADD'A').

The elevation is generally divided into three sections: the eastern prayer hall, the western prayer hall, and the iwan. Identifying the endpoints of the iwan clarifies this tripartite division. The geometric analysis of this spatial division reveals that to determine the width of the iwan (MM′), the half-square principle has been applied (using the 2 and √5 dimension ratio) (Golombek & Wilber, 1995, p. 200). To find point M, an arc is drawn from center A with a radius equal to AB′. This arc intersects the elevation line (AD) at point M. Due to the symmetry of the plan, point M′ is found in the same manner.

To determine the location of segment NN′, which marks the main entrance to the prayer hall, an arc is drawn from center A with radius AA′, intersecting the elevation line (AD) at point N. Similarly, an arc from center A′ with radius A′O intersects the elevation at point N′. Thus, the width of the main entrance to the prayer hall is defined. Given the symmetry of the building plan, other corresponding points are determined using the same method (Figure 13).

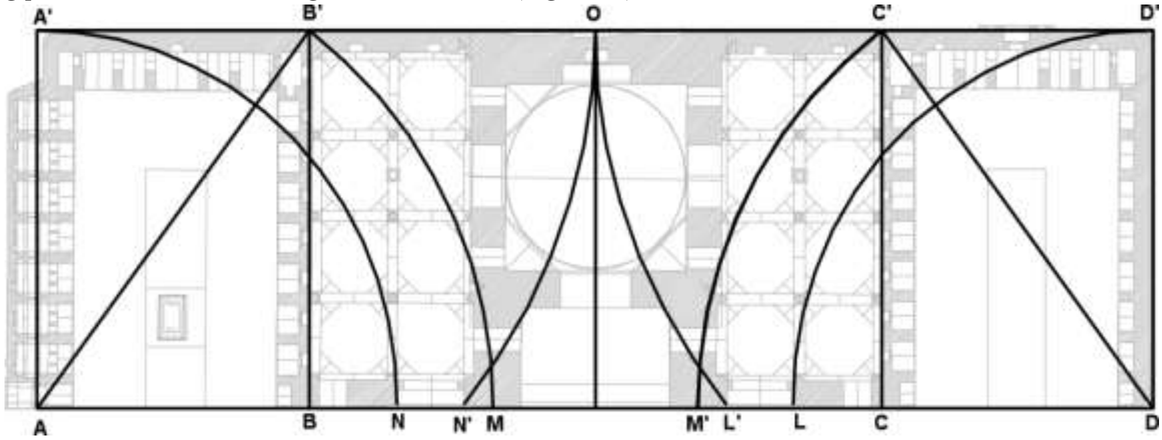

Figure 13. Geometric Analysis of the Plan and Identification of Corresponding and Matching Points with the Elevation

By transferring the obtained points from the plan to the elevation, the other proportions in the elevation are also determined (Figure 14). The vertical lines passing through points M and M' in the elevation not only specify the width of the iwan but also define the symmetry axis of the minarets. To find the height of the five- or seven-segment vault of the iwan (point O'), two arcs are drawn centered at M and M' with radii equal to the height of the iwan; these arcs intersect at point O', which marks the maximum height of the vault. Once the height and width of the vault are specified, the architect proceeds to draw the arch. Similarly, two arcs centered at M and M' with radius MM' intersect at point O'', which determines the height of the decorative ceiling (karbandi). The height of point O''' is equal to half the distance between O' and O''.

To locate point O1, two arcs are drawn centered at points K and P with radii KK' and PP', intersecting at O1. Points O1' and O1'' are then measured upwards and downwards, respectively, from O1 by the height of the iwan's pediment (SS') and half that amount. The ratio of the height of the iwan to the height of the prayer hall is 2.23. Thus, in the geometry of this section, the half-square principle is used, where the ratio between elements is approximately 2.236.

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To find the length BM in the elevation, a square measuring 70×53 (with a 1:3 ratio) is drawn, and then an arc is drawn centered at C with a radius equal to the diagonal of the square. This method allows verification of the accuracy of the dimension previously established in the plan.

To confirm and prove the correctness of the correspondence between the height proportions in the elevation and the plan, arcs are drawn from key height-change points in the elevation centered at C; these arcs intersect the elevation line at points consistent with the plan's proportions. Thus, the conformity of elevation proportions to those of the plan is demonstrated, showing the architect's skill in integrating and matching the geometry of the plan and elevation during design.

Geometry, as an inseparable principle of both structure and elevation, has always been carefully considered. Ultimately, it creates the necessary harmony between structure and elevation, which is clearly evident and proven in this building.

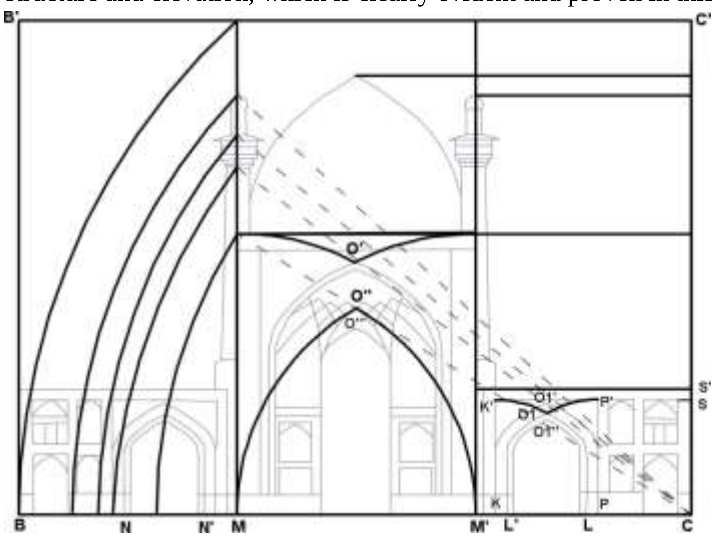

Figure 14. **Geometric alignment of the plan with the elevation using common points from the plan.**

FINDINGS

This study reveals that although past architects did not possess the modern capabilities and tools for structural calculations, they skillfully applied practical geometry principles to create durable and magnificent buildings that have stood firm for centuries. The key to their success lies in the inseparable link between theoretical geometry and practical geometry; meaning that precise, fully calculated geometric designs (theoretical geometry) form the foundation for accurate, stable, and well-executed construction (practical geometry).

The research emphasizes that achieving long-term stability and architectural "eternity" required architects to have a full mastery of geometric knowledge. These architects were fully aware of this necessity and mastered this knowledge at the highest level, applying it in both design and construction. The investigations show that the use of geometry and proportions in these buildings was not merely decorative; rather, geometry constituted the main framework and essence of both the design and structural systems.

The logical and precise proportional relationships across all dimensions and components of the building indicate that no measurements or ratios were chosen arbitrarily. All dimensions and elements were designed and implemented cohesively and harmoniously based on geometric principles. This complete integration of geometry and dimensions clearly proves the study's hypothesis that architects used geometry and proportions as fundamental keys to connect the building's structure with the conceptual ideas in their minds.

One of the most significant points is the extensive and intelligent use of the golden ratio (phi). Recognized as one of the most aesthetically pleasing and harmonious proportions, the golden ratio is prominently applied in the design of domes, shells, and internal spaces. This has resulted in a visual harmony and aesthetic perfection that contributes both to the stability and the captivating beauty of the structure.

CONCLUSION

In general, it can be concluded that theoretical and practical geometry have always been interconnected and complement each other. The architect's proficiency in theoretical geometry has had a direct impact on implementing practical geometry in creating discontinuous double-shell domes and is considered the main influencing factor on the form and construction technology of these domes. Understanding the process of theoretical and practical geometry, by providing deep insight into the construction technology of these domes, today helps to further develop the necessary knowledge for their reconstruction or restoration based on technical expertise and information obtained about their creation methods. This importance becomes even more evident when dealing with buildings that have lost parts and lack sufficient documentation for reconstruction or restoration.

Another outcome of this research is the revival of a portion of indigenous knowledge related to the design and construction of discontinuous double-shell domes, which was revealed through reverse engineering of the design and hidden proportions within the structure of the valuable Abbasi Jame Mosque dome. Since this indigenous knowledge follows geometric and mathematical principles, it is expected that the applicability of the principles revealed in this research to other similar buildings will yield comparable results, which can be explored in future studies. Furthermore, this indigenous knowledge can today be useful in the design and construction of modern shell structures.

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