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Fuzzy Vertex Range Labeling of Some Graph Families

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ABSTRACT: The main objective of this paper is to introduce fuzzy vertex range labeling and look at this for some graph families subject to suitable conditions. In this article, the authors have introduced a new idea in fuzzy graph labeling called fuzzy vertex range labeling. A graph G that can be assigned values as the difference between the maximum and minimum values in fuzzy graph labeling strategy and if all the vertex values are distinct, is called fuzzy vertex range labeling. If G admits fuzzy vertex range labeling then G is called fuzzy vertex range graph. The authors have explored fuzzy vertex range labeling on Fan graph, Double fan graph, Wheel graph and Double wheel graph.

KEYWORDS: Double Fan Graph, Double Wheel Graph, Fuzzy Vertex Range Labeling, Fan Graph, Range Labeling, Wheel Graph.
MSC Classification: 05C72, 05C78, 94D05

1. INTRODUCTION

Based on Zadeh's 1 fuzzy relation (1965), Kaufmann 2 introduced Fuzzy Graph in 1973 and its structure was developed by Azriel Rosenfeld 3 (1975). An increasing number of real-time system models are using Fuzzy Graph theory, where the system's intrinsic information level differs with varying degrees of accuracy. A. Nagoor Gani and D. Rajalaxmi 4 studied Fuzzy Labeling Graphs by discussing their properties. Labeling technique are increasingly important in Fuzzy models since their purpose is to minimize the discrepancies between the conventional numerical methods employed in Engineering and Sciences. Range Labeling in Graph theory was first introduced and developed by R. Jahir Hussain and J. Senthamizh Selvan 5 who applied this concept to some graphs. The new thought called fuzzy range labeling was first proposed by S. Ramya et al. 6,7,8. R. Jebesty Shajila, S. Vimala 9; K. Ameenal Bibi, M. Devi 10 had debated about fuzzy vertex graceful labeling on wheel and fan; double fan and double wheel graphs. Also, R. Shanmugapriya, P.K. Hemalatha, M. Suba 11 had discussed fuzzy vertex graceful labeling on double fan graph and double wheel graph.

In addition to that, this paper is the further contribution on fuzzy graph labeling. "Fuzzy Vertex Range Labeling of Some Graph Families".

2. PRELIMINARIES

Definition 2.1 5: Range Graph

Let $G=(V,E)$ be a graph with n vertices. A bijection on $f:V\to\{1,2,...,n\}$ is called a Range Labeling if for each edge E is distinct and E is defined by

$$f^*(E)=Maximum\ value\,(v_k,v_{k+1})-Minimum\ value\,(v_k,v_{k+1}).$$

If a graph G admits range labeling, we say G is a range graph.

Definition 2.2 12: Fuzzy Graph

A fuzzy graph $G=(V,\sigma,\mu)$ is a triple consisting of a non-empty set V together with a pair of functions $\sigma:V\to[0,1]$ and $\mu:E\to[0,1]$ such that for all $x,y\in V$, $\mu(xy)\le\sigma(x)\wedge\sigma(y)$ where σ denotes the fuzzy vertex set of G and μ denotes the fuzzy edge set of G.

Definition 2.3 4: Fuzzy Labeling Graph

The bijective functions $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ such that the membership values of edges and vertices are distinct and $\mu(u,v)<\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$, is called fuzzy labeling. A graph which admits fuzzy labeling is called a fuzzy labeling graph.

Definition 2.4: Fuzzy Vertex Range Graph

The bijective functions $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ subject to the conditions $\mu(u,v)<\sigma(u)\wedge\sigma(v)$ and $\mu(u,v)=Max.\,value\,(u,v)-Min.\,value\,(u,v)$; for all $u,v\in V$ such that the membership values of edges $\mu(u,v)>0$ and all the membership values of vertices are distinct then it is called fuzzy vertex range labeling. A graph that admits fuzzy vertex range labeling is called fuzzy vertex range graph.

Definition 2.5 15: Fan Graph

A fan graph $F_{m,n}$ is defined as the graph join $\overline{K}_m+P_n$ where $\overline{K}_m$ is the empty graph on m vertices and P_n is the path graph on n vertices.

The case $m=1$ corresponds to the usual *fan graphs*, while $m=2$ corresponds to the *double fan graphs*, etc.

Definition 2.6 16: Wheel Graph

A wheel graph W_n is a graph with n vertices $(n\ge 4)$ formed by connecting a single vertex to all vertices of an $n-1$ cycle.

Definition 2.7 17: Double wheel graph

A double wheel graph DW_n is a graph defined by $2C_n+K_1$. That is, a double wheel graph is a graph obtained by joining all vertices of the two disjoint cycles to an external vertex (known as hub).

3. RESULTS

Theorem: 3.1

Every fan graph $F_{1,n}$ is a fuzzy vertex range graph.

Proof:

When $m=1$, let us take the usual fan graph $F_{1,n}$'s centre vertex be y and the other vertices be x_i $(i=1\text{ to }n)$.

A fan in a fuzzy graph has two vertex sets Y and X_n with $|Y|=1$ and $|X_n|>1$ such that $\beta(y,x_i)>0$, where $i=1\text{ to }n$ and $\beta(x_i,x_{i+1})>0$ where $i=1\text{ to }n-1$.

If the fan graph admits fuzzy vertex range labeling then

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$$\beta(y, x_{n+1}) - \beta(y, x_n) = \beta(x_n, x_{n+1}) \text{ where } n = 1, 2, 3, \ldots$$

Illustration: 3.1.1

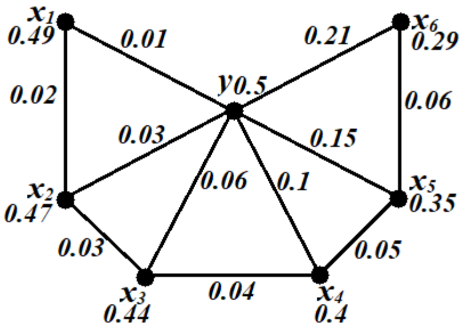

Fig. 1. Fuzzy Vertex Range Labeling for $F_{1,6}$

Case (i): $\alpha(y)$ value starts from $\frac{n-1}{10}$

Here,

$\beta(y, x_1) = 0.01$

$\beta(y, x_2) = \beta(y, x_1) + 0.02 = 0.03$

$\beta(y, x_3) = \beta(y, x_2) + 0.03 = 0.06$

$\beta(y, x_4) = \beta(y, x_3) + 0.04 = 0.1$ etc.,

Therefore $\beta(y, x_{n+1}) = \beta(y, x_n) + (0.01)(n+1)$ where $n = 1, 2, 3, \ldots$

ie., $\beta(y, x_{n+1}) - \beta(y, x_n) = (0.01)(n+1)$ where $n = 1, 2, 3, \ldots$

Also, $\beta(x_1, x_2) = 0.02$; $\beta(x_2, x_3) = 0.03$; $\beta(x_3, x_4) = 0.04$ etc.,

Therefore $\beta(x_n, x_{n+1}) = (0.01)(n+1)$ where $n = 1, 2, 3, \ldots$

So $\beta(y, x_{n+1}) - \beta(y, x_n) = \beta(x_n, x_{n+1})$ where $n = 1, 2, 3, \ldots$

Case (ii): $\alpha(y)$ value starts from $\frac{n-1}{100}$

Here,

$\beta(y, x_1) = 0.001$

$\beta(y, x_2) = \beta(y, x_1) + 0.002 = 0.003$

$\beta(y, x_3) = \beta(y, x_2) + 0.003 = 0.006$

$\beta(y, x_4) = \beta(y, x_3) + 0.004 = 0.01$ etc.,

Therefore $\beta(y, x_{n+1}) = \beta(y, x_n) + (0.001)(n+1)$ where $n = 1, 2, 3, \ldots$

ie., $\beta(y,x_{n+1})-\beta(y,x_n)=(0.001)(n+1)$ where $n=1,2,3,\ldots$

Also, $\beta(x_1,x_2)=0.002$; $\beta(x_2,x_3)=0.003$; $\beta(x_3,x_4)=0.004$ etc.,

Therefore $\beta(x_n,x_{n+1})=(0.001)(n+1)$ where $n=1,2,3,\ldots$

So $\beta(y,x_{n+1})-\beta(y,x_n)=\beta(x_n,x_{n+1})$ where $n=1,2,3,\ldots$

Since all the membership values of vertices are distinct and satisfies the conditions of fuzzy range labeling, every fan graph $F_{1,n}$ is a fuzzy vertex range graph.

Theorem: 3.2

Every double fan graph $F_{2,n}$ is a fuzzy vertex range graph.

Proof:

When $m=2$, let us take the double fan graph $F_{2,n}$'s central vertices be y and y^* and the other vertices be x_i $(i=1 \text{ to } n)$.

A double fan in a fuzzy graph has two vertex sets (Y,Y^*) and X_n with $|Y|=1$, $|Y^*|=1$ and $|X_n|>1$ such that $\beta(y,x_i)>0$ and $\beta(y^*,x_i)>0$, where $i=1 \text{ to } n$ and $\beta(x_i,x_{i+1})>0$ where $i=1 \text{ to } n-1$

If the double fan graph admits fuzzy vertex range labeling then

$$\beta(y,x_{n+1})-\beta(y,x_n)=\beta(x_n,x_{n+1}) \text{ where } n=1,2,3,\ldots$$

$$\beta(y^*,x_{n-1})=\beta(y^*,x_n)-\beta(x_{n-1},x_n)$$

$$ie., \ \beta(y^*,x_n)-\beta(y^*,x_{n-1})=\beta(x_{n-1},x_n) \text{ where } n=1,2,3,\ldots$$

Also, $\alpha(x_n)=\alpha(y)-\beta(y,x_n)$ or $\alpha(x_n)=\alpha(y^*)-\beta(y^*,x_n)$.

Illustration: 3.2.1

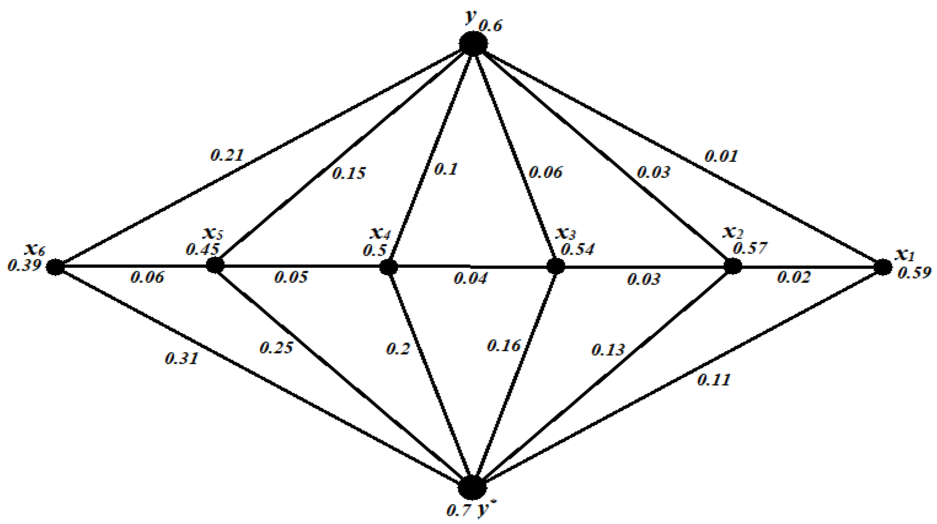

Fig. 2. Fuzzy Vertex Range Labeling for $F'_{2,6}$

Case (i):

$\alpha(y)$ value starts from $\frac{n}{10}$

Here,

$\beta(y,x_1)=0.01$

$\beta(y,x_2)=\beta(y,x_1)+\beta(x_1,x_2)=0.03$

$\beta(y,x_3)=\beta(y,x_2)+\beta(x_2,x_3)=0.06$

$\beta(y,x_4)=\beta(y,x_3)+\beta(x_3,x_4)=0.1$ etc.,

Therefore $\beta(y,x_{n+1})-\beta(y,x_n)=(0.01)(n+1)$ where $n=1,2,3,...$

Also, we have $\beta(x_1,x_2)=0.02$; $\beta(x_2,x_3)=0.03$; $\beta(x_3,x_4)=0.04$ etc.,

Therefore $\beta(x_n,x_{n+1})=(0.01)(n+1)$ where $n=1,2,3,...$

Hence $\beta(y,x_{n+1})-\beta(y,x_n)=\beta(x_n,x_{n+1})$ where $n=1,2,3,...$ and also $\alpha(x_n)=\alpha(y)-\beta(y,x_n)$.

$\alpha(y^*)$ value starts from $\frac{n+1}{10}$

Here, $\beta(y^*,x_4)=\alpha(y^*)-\alpha(x_4)=0.2$

$\beta(y^*,x_3)=\beta(y^*,x_4)-\beta(x_3,x_4)=0.16$

$\beta(y^*,x_2)=\beta(y^*,x_3)-\beta(x_2,x_3)=0.13$

and so on. Therefore $\beta(y^*,x_n)-\beta(y^*,x_{n-1})=(0.01)(n)$ where $n=1,2,3,...$

Also, we have $\beta(x_1,x_2)=0.02$; $\beta(x_2,x_3)=0.03$; $\beta(x_3,x_4)=0.04$ etc.,

Therefore $\beta(x_{n-1},x_n)=(0.01)(n)$ where $n=1,2,3,...$

Hence $\beta(y^*,x_n)-\beta(y^*,x_{n-1})=\beta(x_{n-1},x_n)$ where $n=1,2,3,...$

and also $\alpha(x_n)=\alpha(y^*)-\beta(y^*,x_n)$.

Case (ii):

$\alpha(y)$ value starts from $n/100$

$\beta(y,x_{n+1})-\beta(y,x_n)=(0.001)(n+1)$ where $n=1,2,3,...$

Also, $\beta(x_n,x_{n+1})=(0.001)(n+1)$ where $n=1,2,3,...$

Thus $\beta(y,x_{n+1})-\beta(y,x_n)=\beta(x_n,x_{n+1})$ where $n=1,2,3,...$

$\alpha(y^*)$ value starts from $(n+1)/100$

$\beta(y^*,x_n)-\beta(y^*,x_{n-1})=(0.001)(n)=\beta(x_{n-1},x_n)$ where $n=1,2,3,...$

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Since all the membership values of vertices are distinct and satisfies the fuzzy range labeling conditions, every double fan graph $F_{2,n}$ is a fuzzy vertex range graph.

Remark: 3.3

In fan graph $F_{m,n}$, $\alpha : Y \to [0,1]$ satisfies the condition that $\alpha(y)$ value starts only from $n-1/10, n/10, n+1/10,$ etc., then the fan graph is a fuzzy vertex range graph.

Theorem: 3.4

For some $n \geq 4$, the wheel graph W_n is a fuzzy vertex range graph.

Proof:

A wheel graph W_n with n vertices exists if only $n \geq 4$.

A wheel in a fuzzy graph has two vertex sets Y and X with $|Y| = 1$ and $|X| > 1$, such that $\beta(y, x_i) > 0$, where $i = 1$ to $n-1$ and $\beta(x_i, x_{i+1}) > 0$ where $i = 1$ to $n-2$.

Let the wheel graph's centre vertex be y and the vertices in the outer cycle be x_i.

Then $\alpha : y \to [0,1]$ and $\alpha : x_i \to [0,1]$ defined by

$\alpha(x_i) = \alpha(y) - \beta(y, x_i)$, where $\beta(y, x_i) = (0.01) \times 2^{i-1}$, $i = 1, 2, \ldots, n-1$ and

$\beta(x_i, x_{i+1}) = \beta(y, x_i)$ where $i = 1, 2, \ldots, n-2$ or $\beta(x_{n-2}, x_{n-1}) = \beta(y, x_{n-2})$.

But $\beta(x_{n-1}, x_1) = \beta(y, x_{n-1}) - \beta(y, x_1)$.

Illustration: 3.4.1

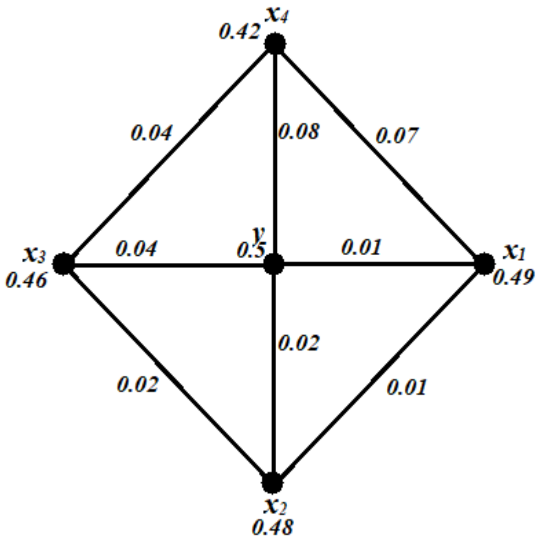

Fig. 3. Fuzzy Vertex Range Labeling for W_5

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Case (i): $\alpha(y)$ starts from $\frac{n-3}{10}$ to $\frac{n+3}{10}$

Here,

$\alpha(x_1)=\alpha(y)-0.01$

$\alpha(x_2)=\alpha(y)-0.02$

$\alpha(x_3)=\alpha(y)-0.04$

$\alpha(x_4)=\alpha(y)-0.08$ etc.,

$\alpha(x_i)=\alpha(y)-(0.01)\times 2^{i-1}$, where $i=1,2,...,n-1$.

ie., $\alpha(x_i)=\alpha(y)-\beta(y,x_i)$, where $\beta(y,x_i)=(0.01)\times 2^{i-1}$, $i=1,2,...,n-1$.

Also, we have $\beta(x_1,x_2)=\beta(y,x_1)$; $\beta(x_2,x_3)=\beta(y,x_2)$; $\beta(x_3,x_4)=\beta(y,x_3)$ etc.,

$\beta(x_i,x_{i+1})=\beta(y,x_i)$ where $i=1,2,...,n-2$ or $\beta(x_{n-2},x_{n-1})=\beta(y,x_{n-2})$.

At the same time, $\beta(x_{n-1},x_1)=\beta(y,x_{n-1})-0.01$. *ie.,* $\beta(x_{n-1},x_1)=\beta(y,x_{n-1})-\beta(y,x_1)$.

Case (ii): $\alpha(y)$ starts from $\frac{n-3}{100}$ to $\frac{n+3}{100}$

Then $\alpha(x_i)=\alpha(y)-\beta(y,x_i)$, where $\beta(y,x_i)=(0.001)\times 2^{i-1}$, $i=1,2,...,n-1$.

Also, $\beta(x_{n-2},x_{n-1})=\beta(y,x_{n-2})$ and $\beta(x_{n-1},x_1)=\beta(y,x_{n-1})-\beta(y,x_1)$.

Since all the vertices values are distinct and satisfies the fuzzy range labeling conditions, W_n (for some $n\geq 4$) is a fuzzy vertex range graph.

Theorem: 3.5

For some $n\geq 4$, the double wheel graph DW_n is a fuzzy vertex range graph.

Proof:

A double wheel graph DW_n with n vertices exists if only $n\geq 4$.

A wheel in a fuzzy graph has two vertex sets Y and X with $|Y|=1$ and $|X|>1$, such that $\beta(y,x_i)>0$, where $i=1$ to n and $\beta(x_i,x_{i+1})>0$ where $i=1$ to $n-1$.

Let the double wheel graph's centre vertex be y, the inner cycle vertices be x_i and the outer cycle vertices be x_i^*.

Then $\alpha: y\rightarrow[0,1]$, $\alpha: x_i\rightarrow[0,1]$ and $\alpha: x_i^*\rightarrow[0,1]$ defined by

For inner cycle, $\alpha(x_i)=\alpha(y)-\beta(y,x_i)$ where $\beta(y,x_i)=(0.001)(i)$; $i=1,2,...,n$ and for outer cycle, $\alpha(x_i^*)=\alpha(y)-\beta(y,x_i^*)$ where $\beta(y,x_i^*)=(0.001)(i+n)$; $i=1,2,...,n$.

For both cycles, the following condition holds,

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$$\beta(x_i, x_{i+1}) = \beta(y, x_i) - \beta(y, x_{i+1}) \text{ where } i = 1, 2, ..., n-1$$

or $\beta(x_{n-1}, x_n) = \beta(y, x_{n-1}) - \beta(y, x_n)$.

Particularly $\beta(x_{n-1}, x_n) = \beta(y, x_{n-1}) - \beta(y, x_n) = 0.001$

But, for getting $\beta(x_n, x_1) = 0.001$, $\beta(x_n, x_1) = \beta(y, x_n) - \beta(y, x_1) - (0.001)(n-2)$.

Illustration: 3.5.1

Case (i): $\alpha(y)$ starts from $\frac{n-1}{100}$

Here, for inner cycle,

$\alpha(x_1) = \alpha(y) - 0.001$

$\alpha(x_2) = \alpha(y) - 0.002$

$\alpha(x_3) = \alpha(y) - 0.003$

$\alpha(x_4) = \alpha(y) - 0.004$ etc.,

$\alpha(x_i) = \alpha(y) - \beta(y, x_i)$, where $\beta(y, x_i) = (0.001)(i)$; $i = 1, 2, ..., n$.

Also, we have $\beta(x_{n-1}, x_n) = \beta(y, x_{n-1}) - \beta(y, x_n) = 0.001$ and

$$0.001 = \beta(x_n, x_1) = \beta(y, x_n) - \beta(y, x_1) - (0.001)(n-2).$$

For outer cycle, $\alpha(x_i^*) = \alpha(y) - \beta(y, x_i^*)$ where $\beta(y, x_i^*) = (0.001)(i+n)$; $i = 1, 2, ..., n$.

Case (ii): $\alpha(y)$ starts from $\frac{n-1}{1000}$

For inner cycle, $\alpha(x_i) = \alpha(y) - \beta(y, x_i)$ where $\beta(y, x_i) = (0.0001)(i)$; $i = 1, 2, ..., n$ and

For outer cycle, $\alpha(x_i^*) = \alpha(y) - \beta(y, x_i^*)$ where $\beta(y, x_i^*) = (0.0001)(i+n)$; $i = 1, 2, ..., n$.

Also, we have $\beta(x_{n-1}, x_n) = \beta(y, x_{n-1}) - \beta(y, x_n) = 0.0001$ and

$$0.0001 = \beta(x_n, x_1) = \beta(y, x_n) - \beta(y, x_1) - (0.0001)(n-2).$$

Since all the vertices values are distinct and satisfies the fuzzy range labeling conditions, DW_n (for some $n \geq 4$) is a fuzzy vertex range graph.

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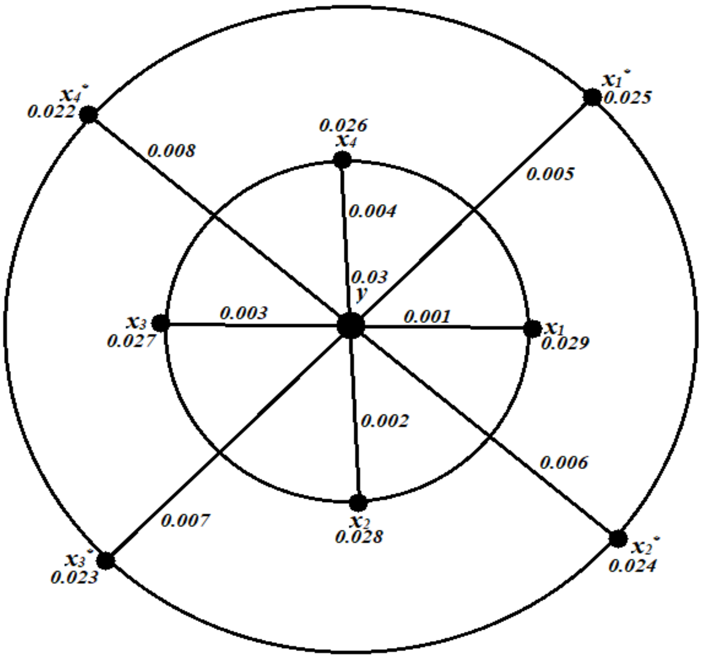

Fig. 4. Fuzzy Vertex Range Labeling for DW_5

4. CONCLUSION

The new idea called Fuzzy Vertex Range Labeling has been introduced and analysed to Fan graph, Double fan graph, Wheel graph and Double wheel graph in this article. Also, we have proved those graphs are fuzzy vertex range graph. We intend to continue this research work on a few more graphs.

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