



Analyse the Representation of Functions Expressed in Arrow Diagrams

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ABSTRACT: The aim of this research is to describe the representation of functions expressed in arrow diagrams, which is an exciting topic. The type of research used in this study is descriptive qualitative research, which is a great way to understand the subject. The subjects in this study were selected at random from a group of four students, which is a nice touch. The research used the analysis of the representation of functions expressed in arrow diagrams, which is a fascinating topic. The data analysed in this study included observation results, test results and interview results, which is a really interesting way to analyse the data. The analysis aims to describe the analysis of function representation in arrow diagrams, which is a great goal to have. The activities carried out in this study were really interesting. Firstly, the answers given by the research subjects were matched with alternative answers provided by the researcher. Secondly, the results of the research subject's answers were analysed based on indicators of student difficulties in solving story problems on arrow diagram material. The results of the arrow diagram representation made by students were then used to create representations of functions in the relations 'multiples of', 'factors of', 'squares of', 'half of', '3 times of', whose set members are numbers and letters. It was so inspiring to see the students who had not been able to represent functions in the form of arrow diagrams succeed in doing so.

KEYWORDS: Arrow Diagrams, Function Representation.

INTRODUCTION

In the field of mathematics, functions are instrumental in depicting relationships between two sets of elements. A comprehensive understanding of the visual representation of these relationships is paramount for the analysis and interpretation of their properties. Arrow diagrams are a particularly efficacious method for visualizing functions. These diagrams employ arrows to map elements of a domain to corresponding elements in a codomain, offering an intuitive and straightforward means to explore the nature of the function. The present article aims to analyse the representation of functions expressed in arrow diagrams, focusing on how they illustrate critical aspects such as injectivity, surjectivity, and bijectivity. By examining these visual representations, one can gain deeper insights into the behavior of functions and their applications in various fields, including computer science, engineering, and data analysis. The discussion will address the strengths and limitations of using arrow diagrams in understanding functions, with a particular focus on their use in educational contexts, where they play an instrumental role in fostering mathematical intuition. The analysis presented in this article aims to facilitate a comprehensive understanding of how arrow diagrams effectively communicate the structure and properties of functions, thereby bridging the gap between abstract concepts and practical visualisation.

RESEARCH METHOD

This study utilizes descriptive qualitative research to ascertain and delineate the outcomes of analyzing the depiction of functions depicted in arrow diagrams. This research followed an initial assessment of students' competencies concerning arrow diagram material, carried out in class VIII. Four students were chosen for this research using the purposive random sample method. The study executed a trial involving textual difficulties related to arrow diagram concepts. The participants were picked according to their problem-solving performance, categorizing them into high, middle, and low achievers. The study employed instruments to facilitate the analysis, including a test aimed at evaluating the representation capability in arrow diagrams, offering a comprehensive examination of the representation of functions depicted in arrow diagrams. The research also employed the examination of function representations depicted in arrow diagrams to enhance the study. The data collection methods utilized in this study comprised: a. Student assessments, employed to analyze student representation. b. Student interviews utilized to

elucidate students' representation analysis. Student documentation utilized in this study to catalog the quantity of photographs and recordings pertinent to testing and interview activities. The execution of photographic documenting and recording is conducted by others. This data is presented as student work organized according to the research subject. This activity presents a compilation of structured and classified data that facilitates the derivation of conclusions and subsequent actions. The analysis of function representation depicted in the arrow diagram for each subject is concluded based on the data presentation.

RESULT AND DISCUSSION

Function Representation Subject 1

The research subjects consisted of 4 students, namely S1, S2, S3 and S4. S1, S2, S3 and S4 are VIII grade students with diverse abilities.

- Arrow Diagram Representation of Student 1.

Student 1 made 3 representations of the function in the arrow diagram. The first representation is a function with domain and codomain

$A = \{1, 2, 3, 4, 5\}$

$B = \{4, 9, 16, 25\}$

The pairs of set members made by students are as follows

- 1 paired with 4
- 2 paired with 4
- 3 paired with 9
- 4 paired with 16
- 5 paired with 25

The relation that students make is "multiples of" Here are the results of the arrow diagram representation by subject 1 in figure 1.

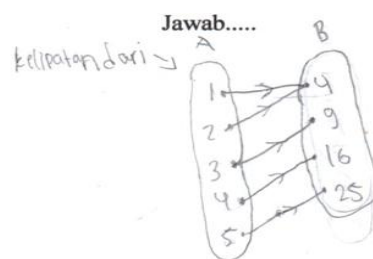


Figure 1. Arrow Diagram Representation by Subject S1

Based on this, the statements that can be concluded are as follows

- 4 multiples of 1
- 4 multiples of 2
- 9 multiples of 3
- 16 multiples of 4
- 25 multiples of 5

The relation "multiples of" made in the first arrow diagram made by students is incorrect. This should be 4 is a "multiple of" 1, 4 is a "multiple of" 2, 9 is a "multiple of" 3, 16 is a "multiple of" 4, 25 is a "multiple of" 5. However, students represent in the arrow diagram with multiples of is less precise.

The second representation is a function with a domain and a codomain.

$A = \{1, 2, 3, 4, 5\}$

$B = \{2, 3, 5, 7\}$

The pairs of set members made by students are as follows

- 1 is paired with 2

- 2 paired with 3
- 3 paired with 3
- 4 paired with 4
- 5 paired with 5

The relation that students make is "factor of". The following is the result of the arrow diagram representation by S1 in Figure 2.

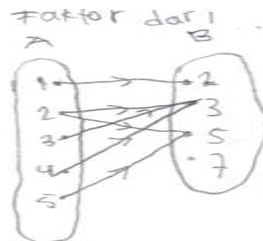


Figure 2. Arrow Diagram Representation by S1

Based on this, the statements that can be concluded are as follows

- 1 factor of 2
- 2 factors out of 3
- 3 factors out of 3
- 4 factors out of 3
- 5 factors out of 5

The relation "factor of" made in the first arrow diagram made by the student is not correct. This is because 2 is not a "factor of" 3. The "factor of" made by students is not correct. This is because 4 is not a "factor of" 3, because 2 cannot be multiplied by an integer to produce 3, 3 is a "factor of" 3, 4 is not a "factor of" 3, because 4 cannot be multiplied by an integer to produce 3, 5 is a "factor of" 5. So what is made in the arrow diagram by students is still incorrect.

The third representation is a function with domain and codomain

$$A = \{0, 1, 4, 9\}$$

$$B = \{0, 1, 2, 3, 4\}$$

The pairs of set members made by students are as follows

- 0 is paired with 0
- 1 is paired with 1
- 4 paired with 2
- 9 paired with 3

The relation that students make is "the square of". The following is the result of the arrow diagram representation by S1 in figure 3.

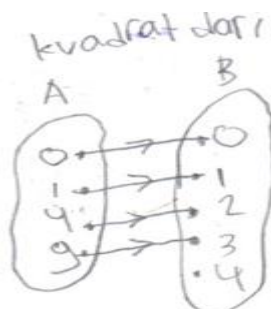


Figure 3. Arrow Diagram Representation by S1

Based on this, the statements that can be concluded are as follows

- 0 squared of 0
- 1 square of 1
- 4 squared of 2
- 9 squared of 3

The "square of" relation made in the first arrow diagram made by students is correct. This is because 0 is the "square of" 0, 1 is the "square of" 1, 4 is the "square of" 2, 9 is the "square of" 3 exactly.

Of the 3 arrow diagrams made by S1, there are 2 correct, less correct arrow diagrams and 1 incorrect arrow diagram. The correct relations made are multiples of and squares of. The less precise relation is the factor of. All the arrow diagrams made represent functions.

Representation of Function in Arrow Diagram by Subject 2

Student 2 made 3 representations of the function in the arrow diagram. The first representation is a function with domain and codomain

$$A = \{2, 3, 4, 5, \}$$

$$B = \{4, 6, 8, 10\}$$

The pairs of set members made by students are as follows

- 2 paired with 4
- 3 paired with 6
- 4 paired with 8
- 5 paired with 10

The relation that students make is "half of". The following is S2's work in Figure 4.

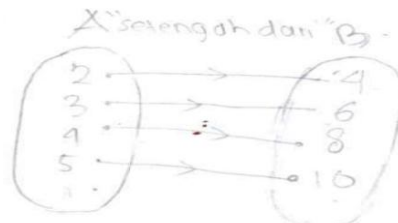


Figure 4. Arrow Diagram Representation of S2

Based on this, the statements that can be concluded are as follows

- 2 half of 4
- 3 half of 6
- 4 half of 8
- 5 and a half out of 10

The relation "half of" made in the first arrow diagram made by students is correct. This is because 2 is "half of" 4, 3 is "half of" 6, 4 is "half of" 8, 5 is "half of" 10. All pairs made in the arrow diagram by students are correct.

The second representation is a function with domain and codomain

$$A = \{6, 9, 15, 24, 27\}$$

$$B = \{2, 3, 5, 8, 9\}$$

The pairs of set members made by students are as follows

- 6 paired with 2
- 9 paired with 3
- 15 paired with 5
- 24 paired with 8
- 27 paired with 9

The relation that students make is "3 times of". The following is S2's work in figure 5,

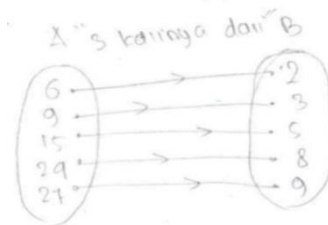


Figure 5. Arrow Diagram Representation of S2

Based on this, the statements that can be concluded are as follows

- 6 "3 times of" 2
- 9 "3 times of" 3
- 15 "3 times of" 5
- 24 "3 times of" 8
- 27 "3 times of" 9

The relation "3 times of" made in the first arrow diagram made by students is appropriate. This is because 6 is "3 times of" 2, 9 is "3 times of" 3, 15 is "3 times of" 5, 24 is "3 times of" 8, 27 is "3 times of" 9 exactly. So everything the student made is correct.

The third representation is a function with domain and codomain

$$A = \{2, 4, 6, 8\}$$

$$B = \{1, 2, 3, 4\}$$

The pairs of set members made by students are as follows

- 2 paired 1
- 4 paired 2
- 6 paired 3
- 8 paired 4

The relation that students make is "2 times of". Here are the results of S2's work in Figure 6.

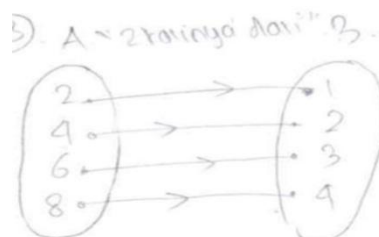


Figure 6. Arrow Diagram Representation by S2

Based on this, the statements that can be concluded are as follows

- 2 "2 times of" 1
- 4 "2 times of" 2
- 6 "2 times of" 3
- 8 "2 times of" 4

The relation "2 times of" made in the first arrow diagram made by students is appropriate. This is because 2 is "2 times of" 1, 4 is "2 times of" 2, 6 is "2 times of" 3, 8 is "2 times of" 4, so the students made are all correct. Of the 3 arrow diagrams made by S2, all are correct. All the arrow diagrams made are representing functions.

Representation of Function in Arrow Diagram by Subject 3

Student 3 made 3 representations of the function in the arrow diagram. The first representation is a function with domain and codomain

$$A = \{2, 6, 8, 9, 15, 17, 21\}$$

$$B = \{3, 4, 5, 7\}$$

The pairs of set members made by students are as follows

- 2 no partner.
- 6 paired with 3
- 8 paired with 4
- 9 paired with 3
- 15 paired with 5
- 17 paired with 3
- 21 paired with 7

The relation that students make is "multiples of". Here are the results of S3's work in Figure 7.

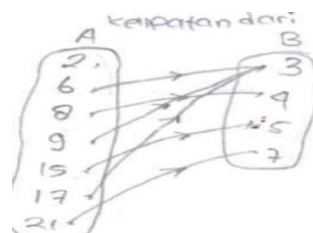


Figure 7. Arrow Diagram Representation by S3

Based on this, the statements that can be concluded are as follows

- 6 multiples of 3
- 8 multiples of 4
- 9 multiples of 3
- 15 multiples of 5
- 17 multiples of 3
- 21 multiples of 7

The relation "multiples of" made in the first arrow diagram made by students is incorrect. Based on the results of the arrow diagram 2 made, there is a number 2 that does not have a pair. So that the representation of the arrow diagram made does not illustrate the function. A function is a special relation that pairs exactly one set member with another set member. Not every relation is a function. The origin member must be paired exactly one to the result member.

The second representation is a function with domain and codomain

$$A = \{6, 9, 15, 21, 24, 27\}$$

$$B = \{2, 3, 5, 8, 9\}$$

The pairs of set members made by students are as follows

- 6 paired with 2
- 9 paired with 3
- 15 paired with 5
- 21 paired with 5
- 24 paired with 8
- 27 paired with 9

The relation that students make is "multiples of". Here are the results of S3's work in Figure 8.

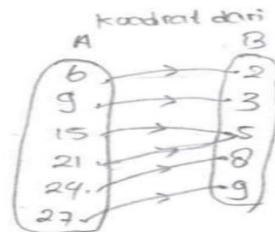


Figure 8. Arrow Diagram Representation by S3

Based on this, the statements that can be concluded are as follows

- 6 squares of 2
- 9 squared of 3
- 15 squares of 5
- 21 squared of 5
- 24 squared of 8
- 27 squared of 9

The "square of" relation made in the first arrow diagram made by students is less precise. This is because 6 is not the "square of" 2, 9 is the "square of" 3, 15 is not the "square of" 5 less precise. 21 is not the "square of" 5, 24 is not the "square of" 8, 27 is not the "square of" 9.

The second representation is a function with domain and codomain

$$A = \{2, 4, 6, 8\}$$

$$B = \{1, 2, 3, 4\}$$

The pairs of set members made by students are as follows

- 2 paired with 1
- 4 paired with 2
- 6 paired with 3
- 8 paired with 4

The relation that students make is "two times of". The following is the result of the arrow diagram representation by S3 in Figure 9.

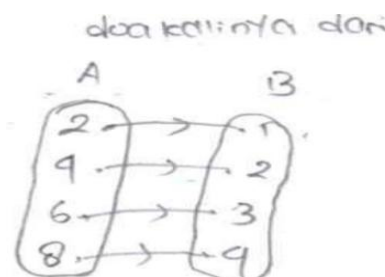


Figure 9. Arrow Diagram Representation by S3

Based on this, the statements that can be concluded are as follows

- 2 "two times of" 1
- 4 "twice as much as" 2
- 6 "twice as much as" 3
- 8 "twice as much as" 4

The relation "two times of" made in the first arrow diagram made by students is correct. This is because 2 is "twice of" 1, 4 is "twice of" 2, 6 is "twice of" 3, 8 is "twice of" 4.

Of the 3 results of the arrow diagram made by S3, there is 1 correct arrow diagram, less precise 2. The right relation made is twice of. Less precise multiples of and squares of. The overall arrow diagram made less precisely represents the function. in the arrow diagram there are 2 arrow diagrams that represent the function and 1 arrow diagram that does not represent the function. The relation made from the correct arrow diagrams is the relation "twice of" and less correct multiples of and squares of.

Representation of Function in Arrow Diagram by Subject 4

Student 3 makes 3 representations of the function in an arrow diagram

The first representation is a function with domain and codomain

$$A = \{1, 2, 3\}$$

$$B = \{a, b\}$$

The pairs of set members made by students are as follows

- 1 paired with b
- 2 paired with a
- 3 paired with a

The relation that students make is to make a relation that is a function. The following is the result of the arrow diagram representation by S4 in figure 10.

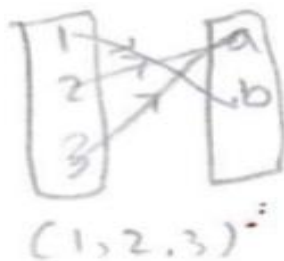


Figure 10. Arrow Diagram Representation by S4

Based on this, the statements that can be concluded are as follows

- 1 paired with b
- 2 paired with a
- 3 paired with a

The relation made in the first arrow diagram made by students is less precise. This is because there is no rule that determines the arrow diagram.

The second representation is a function with domain and codomain

$$A = \{6, 9, 15, 21, 24, 27\}$$

$$B = \{2, 3, 5, 8, 9\}$$

The pairs of set members made by students are as follows

- 6 paired with 2
- 9 paired with 3
- 15 paired with 5
- 24 paired with 8
- 27 paired with 9

The relation that students make is to make a relation that is a function. The following is the result of the arrow diagram representation by S4 in figure 11.

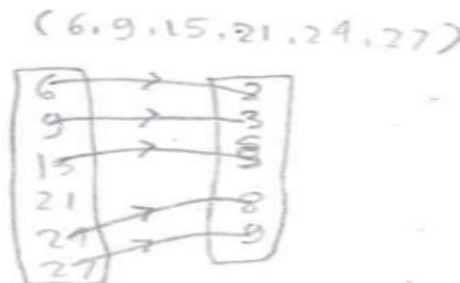


Figure 11. Arrow Diagram Representation by S4

The relation that students make is to make a relation that is a function. The following is the result of the arrow diagram representation by subject 4 in figure 11.

- 6 paired with 2
- 9 paired with 3
- 15 paired with 5
- 24 paired with 8
- 27 paired with 9

The relation made in the first arrow diagram made by students is less precise. This is because there is no defining rule in the arrow diagram.

The third representation is a function with domain and codomain

$$A = \{0, 4, 9\}$$

$$B = \{0, 4, 3, 2, 1\}$$

The pairs of set members made by students are as follows

- 1 is paired with 1
- 4 paired with 4
- 9 paired with 3

The relation that students make is to make a relation that is a function. The following is the result of the arrow diagram representation by S4 in figure 12.

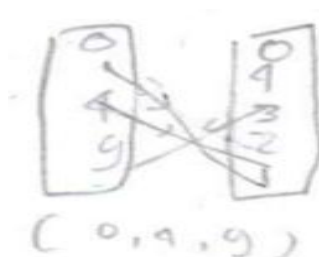


Figure 12. Arrow Diagram Representation by S4

The relation that students make is to make a relation that is a function. The following is the result of the arrow diagram representation by subject 4 in figure 4.1.6

- 1 is paired with 1
- 4 paired with 4
- 9 paired with 3

The relation made in the first arrow diagram made by students is less precise. This is because there is no rule that determines the arrow diagram.

Of the 3 arrow diagrams made by S4, all are less precise. Because there are still does not yet have a pair so that the relation made by S4 is less precise, and different

with other subjects, S4 has not yet determined the relation and there are members that are numbers and letters.

DISCUSSION

Function is an important mathematical concept. Based on the results of research taught in junior high school (Raharjo, 2020). The function representation made by S2 and S3 in the arrow diagram is a function. The representation of the function made by students S2, S3 in this arrow diagram is "3 times of" as in the picture as follows.

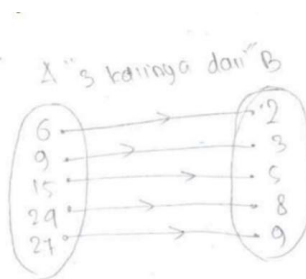


Figure 13. Arrow Diagram Representation

The arrow diagram depicted in Figure 13. already represents a function. This is because all members of set A have one exact match in the members of set B. The relation that can be made is from set $A = \{6, 9, 15, 24, 27\}$ to $B = \{2, 3, 5, 8, 9\}$ which is "3 times of" After learning the relation and function material, students are expected to understand the concepts of relation and function; present relations in the form of cartesian diagrams arrow diagrams and sets of consecutive pairs; understand solutions using function values; understand and graph functions; and solve. Mathematical communication is a way for students to express their mathematical ideas either orally, in writing, drawings, diagrams, expressing objects, presenting in algebraic form, or using mathematical symbols (NCTM, 2000).

Representation in mathematical communication can help the process of refining the understanding of mathematical ideas, and help build the meaning and permanence of an idea. If students are challenged to think and reason about mathematics, and communicate the results of their thinking orally and in writing, then with the help of representations they can gain a clearer and more convincing understanding. The mapping function of each part of a set (called the domain) to another part of the set (called the codomain).

The representation of the function made by S1 and S4 in the arrow diagram illustrates the function. The representation raised by students is an expression of mathematical ideas or ideas that students display in their efforts to find a solution to the problem they are facing. Based on the results of the representation of the function made by students S1, S4 in this arrow diagram is a "factor of" as in the picture as follows.

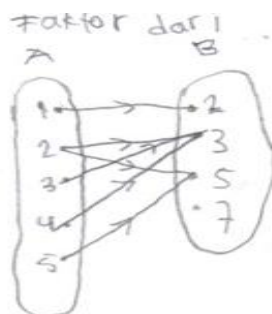


Figure 14. Arrow Diagram Representation

The arrow diagram depicted in Figure 14. does not represent a function. This is because it is not correct, because the domain area A has more than one pair, the relation made from the set $A = \{1, 2, 3, 4, 5\}$ to $B = \{2, 3, 5, 7\}$ is "factor of". Mathematical representation ability is a basic ability that is very important in learning mathematics. (Dewiyani 2010).

representation is a configuration that can represent something else in some way. Mathematical representation ability is a basic ability that helps students express mathematical ideas in various ways, namely in the form of pictures, tables, graphs, numbers, mathematical symbols and writing.

The representation of the function made by students S2, S3 in this arrow diagram is "2 times 3". The arrow diagram illustrates the function. as in the picture as follows.

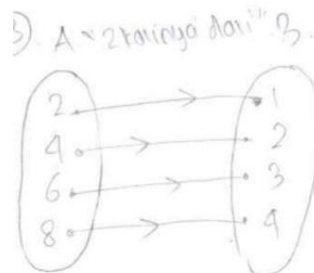


Figure 15. Arrow Diagram Representation

The arrow diagram depicted in Figure 15 already represents a function. This is because all members of set A have exactly one pair of members of set B. The relation that can be made is from the set $A = \{ 2, 4, 6, 8 \}$ to $B = \{ 1, 2, 3, 4 \}$ is A "2 times than B". Although mathematical representation ability is one of the abilities that students need to have, but in reality there are still many teachers who put aside the ability of mathematical representation. This is in line with the opinion of Hudiono (2005:4) which states that according to teachers, mathematical representations in the form of graphs, tables, and drawings are only a complement to learning and teachers rarely pay attention to the development of students' mathematical representation skills. Representation is a model or form used to represent a situation or problem in order to facilitate the search for solutions. Representation is inseparable in learning mathematics.

Although not explicitly listed in the objectives of mathematics learning in Indonesia, the importance of representation is implicit in the objectives of problem solving and mathematical communication, because to solve mathematical problems, the ability to create mathematical models and interpret their solutions is required, which is an indicator of representation.

The representation of the function made by the student, in this arrow diagram is "the square of". The arrow diagram illustrates the function. as in the picture as follows.

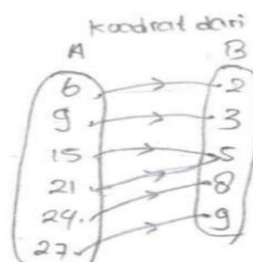


Figure 16. Arrow Diagram Representation

The arrow diagram depicted in Figure 16. already represents a function. This is because all members of set A have exactly one pair of members of set B. The relation that can be made is from set $A = \{ 6,9,15,21,24,27 \}$ to $B = \{ 2,3,5,8,9 \}$ is "the square of". Although mathematical representation ability is one of the abilities that students need to have, but in reality there are still many teachers who put aside the ability of mathematical representation. This is in line with the opinion of Hudiono (2005:4) which states that according to teachers, mathematical representations in the form of graphs, tables, and drawings are only a complement to learning and teachers rarely pay attention to the development of students' mathematical representation skills.

Mathematical representation is one of the high-level mathematical thinking skills, consisting of internal and external representations. Internal representations cannot be observed because they exist mentally, while mathematical communication is an external representation.

The representation of functions made by students, in this arrow diagram is already describing the function. as in the picture as follows.

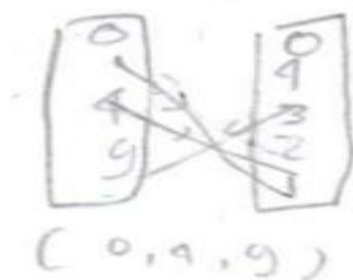


Figure 17. Arrow Diagram Representation

The arrow diagram depicted in Figure 17. does not represent a function. This is because all members of set A have no pairs and members of set B also have no pairs. The relations made by students do not use rules. The relation that can be made is from the set $A = \{0,4,9\}$ $B = \{0,4,3,2,1\}$. Mathematical representation ability is one of the general objectives of learning mathematics at school. This ability is very important for students and is closely related to communication and problem solving skills. A person needs representation in the form of pictures, graphs, diagrams, and other forms of representation to be able to communicate something (Lette & Manoy, 2019).

The representation of the function made by S3 in the arrow diagram is a function. The representation of the function made by S3 students in this arrow diagram is "multiples of" as in the picture as follows.

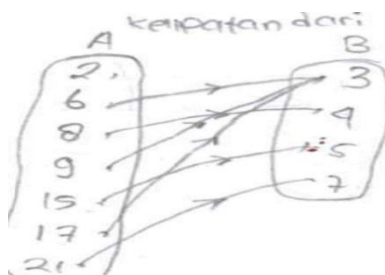


Figure 18. Arrow Diagram Representation

The arrow diagram depicted in Figure 18 already represents a function. This is because all members of set A have one exact match in the members of set B. The relation that can be made is from set $A = \{2,6,8,9,15,21\}$ to $B = \{3,4,5,7\}$ is "multiples of". According to Lestari and Yudhanegara (2015: 83), mathematical representation ability is the ability to re-present notations, symbols, tables, images, graphs, diagrams, equations or other mathematical expressions into other forms. mathematical representation ability encourages students to find and create tools or ways of thinking in communicating mathematical ideas. Mathematical representation ability is one of the abilities that support other competencies.

It can be seen in the problems that S1, S2, S3, and S4 have done regarding the arrow diagram that students are able to draw and represent problem solving. However, there are some subjects who still do not understand the meaning of the problem in solving it by drawing first. The subject solves it by using the arrangement of making the arrow diagram, then each step produced by the subject is also described in the form of representation.

When understanding problems, there are differences in reading problems by students repeatedly with each learning style they have (Arum, 2017), where problems regarding understanding can be found in Relation and Function material. In the observation results, it is still found that students have difficulty in distinguishing between relations and functions and students are still confused in understanding the concept of functions.

Surianto (2014) explained that students still find it difficult when solving problems related to relation and function material, for example students find it difficult when working on problems from understanding what is known in the problem and the questions in the problem. Students assume that the material of relations and functions is the same, which is about the rules connecting the members of each set A with members of set B.



CONCLUSION

Based on the results of the arrow diagram representations made by students, students make representations of functions in the relations "multiples of", "factors of", "squares of", "half of", "3 times of", whose set members are numbers and letters. There are students who have not been able to represent functions in the form of arrow diagrams. Arrow diagrams are recommended as an initial method of teaching function concepts to students, before introducing more abstract symbolic or graphical representations. The use of this tool is considered capable of providing a strong foundation in mathematical understanding.

ADVICE

1. For teachers
Learning using the function representation method can be used as an alternative learning method in the classroom. The function representation method can also be used on arrow diagram material.
2. For learners
Learners should be more independent and active in studying, not just relying on subject teachers.
3. For researchers

Future research should find the right methods and strategies to improve students' function representation skills

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