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Learning Activities in Mathematics Education: Application of the PBL Model and RME Approach in the Power Dominating Set for Solving Electricity Network Optimization Problems to Enhance Students' Critical Thinking Skills

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ABSTRACT: Mathematics education has a significant role in developing students' critical thinking skills, especially in dealing with complex problems. In an effort to support this, this study aims to evaluate the effectiveness of applying the Problem-Based Learning (PBL) learning model combined with the Realistic Mathematics Education (RME) approach to learning the concept of Power Dominating Set (PDS). The PDS concept, which is part of graph theory, is used to solve power network optimization problems by minimising the number of control points. This is expected to improve the efficiency of resource use and the overall performance of the power grid. This research method involves the application of the PBL model, where students are actively involved in the learning process through real context-based problem solving, as well as the RME approach that connects abstract mathematical concepts with realistic situations. The learning process is focused on mastering the concept of PDS both from the theoretical side and its application in power network optimisation. The results showed that the combination of the PBL model and the RME approach significantly improved students' understanding of the PDS concept, both in terms of mathematical abstraction and its application in practical contexts. In addition, this approach proved effective in developing students' critical thinking skills, especially in analysing and solving complex power network optimisation problems. The discussion of the results of this study highlights that learning strategies that integrate PBL and RME are able to provide deeper and more relevant learning experiences for students. This approach not only helps students understand complex mathematical concepts, but also trains them to apply the knowledge in real situations. The implications of this research provide important insights for educators to adopt innovative learning methods that can improve the quality of mathematics learning, especially in teaching applicable concepts such as Power Dominating Set.

KEYWORDS: Critical thinking skills, Electricity network optimization, Problem-Based Learning, Power Dominating Set, Realistic Mathematics Education.

1. INTRODUCTION

Mathematics education has an important role in building students' critical thinking skills, especially in an era that demands complex problem solving and application of knowledge in real contexts. In the context of electricity network management, the concept of Power Dominating Set (PDS) is one of the relevant mathematical approaches to solve optimization problems. The connection between mathematical theory and its application in the real world is an important foundation to improve students' conceptual understanding. Therefore, an innovative learning approach is needed to bridge mathematical theory with its implementation, while improving students' critical thinking skills.

Learning mathematics in the modern era demands integration between theory and real applications to increase the relevance of science to the needs of industry and daily life. One of the growing research areas is the application of Problem-Based Learning (PBL) model and Realistic Mathematics Education (RME) approach in mathematics learning. PBL has been widely recognised as an effective method for improving critical thinking skills through contextual problem solving. Meanwhile, RME emphasises the use of real contexts as a starting point for learning, allowing students to connect abstract concepts with real-life situations.

In the context of power network optimisation, the concept of Power Dominating Set (PDS) is an interesting topic due to its important role in ensuring efficient network monitoring with limited resources. Although research on PDS has been widely

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conducted in the field of graph theory and its applications, the application of this concept in mathematics learning through PBL and RME to improve students' critical thinking skills is still rarely studied in depth.

Critical thinking skills consist of five main indicators that reflect various analytical and evaluative aspects (Arif et al., 2020). The first indicator is basic clarification, which includes the ability to formulate critical questions, analyse arguments, and ask and answer clarifying questions to understand a problem in depth. The second indicator, providing bases for a decision, focuses on the ability to consider the credibility of a source of information and to observe and evaluate the results of observations properly. The third indicator, inference, involves deduction and induction processes to draw logical conclusions, as well as considering the value and consequences of decisions taken.

Furthermore, advanced clarification includes the ability to identify terms precisely, consider relevant definitions, and refer to assumptions that are not explicitly stated. The last indicator, supposition and integration, highlights the ability to logically consider elements such as premises, reasons, assumptions and positions, and integrate all critical thinking skills to make and defend solid decisions. These five indicators are important guides in measuring and developing critical thinking skills in various learning contexts.

Research related to the application of Problem-Based Learning (PBL) in mathematics education has shown significant potential in improving students' critical thinking skills and mathematical reasoning ability. Arviana and Dewi (2018) found that the PBL model effectively encourages students to develop critical thinking skills through the exploration of complex and contextualised mathematical problems, making it relevant for learning orientated towards solving real problems. Dahl (2018) criticised the implementation of PBL in higher mathematics education, highlighting the importance of appropriate problem design to encourage active and effective student participation, especially in understanding abstract mathematical concepts. Furthermore, Mandasari (2021) asserted that the PBL model not only improves critical thinking skills, but also students' mathematical reasoning skills, especially in concept-based learning and real contexts, as shown in a study at a physics conference. Overall, these studies reinforce the relevance of PBL as an effective approach to improving students' conceptual understanding and critical thinking skills in mathematics education.

State-of-the-art research related to the application of the Problem-Based Learning (PBL) model with the Realistic Mathematics Education (RME) approach shows significant effectiveness in improving various aspects of mathematics learning, such as problem-solving ability, mathematical literacy, and students' reasoning skills. Kurniawati et al. (2023) highlighted the effectiveness of PBL and RME integration in improving problem-solving skills, while Mariani (2019) and Waluya & Mariani (2016) underlined that this approach, especially with the help of digital platforms such as e-Schoology and Edmodo, strengthens students' mathematisation skills. Maslihah et al. (2021) showed how the combination of PBL and RME can improve students' learning independence and mathematical literacy, while Rewah et al. (2021) focused on developing PBL and RME-based learning tools on the Pythagorean theorem which showed positive results in the context of innovative learning. Susanti & Rustam (2018) proved that RME and PBL models are effective in improving junior high school students' mathematical reasoning skills, which is reinforced by the findings of Yuanita et al. (2018) which showed that mathematical representation mediates the relationship between mathematical confidence and problem solving ability. Umar & Zakaria (2022) added that the manipulative media-based RME method was highly effective in improving the problem-solving skills of primary school students, indicating the broad potential of this approach at different levels of education. Overall, these studies support the relevance of PBL and RME as innovative approaches in improving students' mathematical skills across different contexts and levels of education.

The study of domination in graphs and its applications, including the concept of Power Dominating Set (PDS), has received extensive attention in the literature. Alikhani et al. (2024) explored total coalitions in graphs that contribute to the generalisation of the domination concept, while Chellali et al. (2020) explored Roman dominance as an extension of the classical theory in this topic. The research of Dorbec et al. (2008) provides a specialised approach to PDS in product graphs, which is relevant for complex network applications. In a practical context, Haynes et al. (2002) showed the importance of dominance in graphs for power grid optimisation, and their recent work (2020, 2023) extends the discussion by focusing on the core concepts and various applications, including power grids. Prabhu et al. (2024) introduced PDS resolution in fractal cubic networks, demonstrating the potential application of graphs to specialised network structures. Saibavani and Parvathi (2023) applied the PDS concept to various types of graphs, highlighting the flexibility of the theory in practical applications. Zhao et al. (2006) noted the theoretical basis of PDS which became a key reference for further research. This study shows that graph dominance, particularly PDS, is a dynamic area with great potential to be applied in network optimisation, including in power grids.

Furthermore, this article offers novelty by combining the PBL learning model and the RME approach in the context of solving electricity network optimization problems based on the concept of Power Dominating Set. Thus, the novelty of this research lies in: (1) the application of theoretical mathematical concepts (PDS) in real context-based learning scenarios through the RME approach. (2) Exploration of the PBL model as a learning medium to improve students' critical thinking skills, especially in understanding and solving optimization problems. (3) Studies that integrate aspects of mathematics education with direct applications in the field of electrical network engineering, which provide new contributions to the development of interdisciplinary learning models.

The objectives of this study are as follows: (1) to apply the Problem-Based Learning (PBL) model in mathematics learning on the concept of Power Dominating Set; (2) to apply the Realistic Mathematics Education (RME) approach to support students' understanding of the concept of Power Dominating Set in the context of power network optimisation; and (3) to apply the PBL model and RME approach to the Power Dominating Set material to improve students' critical thinking skills.

2. METHOD

This research is a qualitative study that aims to analyze the application of the Problem-Based Learning (PBL) model and the Realistic Mathematics Education (RME) approach in learning the topic of Power Dominating Set to solve power network optimization problems and improve students' critical thinking skills. This research uses an RBL-STEM-based learning activity development framework that refers to the 4D model, namely define, design, develop, and disseminate.

The Define stage involves identifying learning needs and formulating PBL and RME-based learning objectives. The Design stage includes the preparation of learning devices, such as syllabus, lesson plans, and student worksheets, as well as the design of research instruments, such as interview, observation, and reflection guidelines. The Develop stage includes the implementation of learning devices in experimental classes and revisions based on feedback during implementation. The Disseminate stage is done by disseminating the research results through seminars or journal publications as well as training to other lecturers. The research data is in the form of qualitative data obtained from observations of the learning process, interviews with students, and student work documents. The instruments used include observation guidelines, interview guides, and reflection sheets. Data collection techniques were carried out through direct observation, in-depth interviews, and analysis of student work documents.

The data collected were analyzed thematically through the steps of data organization, theme coding, categorization, and conclusion drawing to answer the research objectives. This research is expected to provide a comprehensive picture of the effectiveness of the application of the PBL model and the RME approach in learning mathematics, especially on the topic of Power Dominating Set, as well as its contribution in improving students' critical thinking skills.

3. RESULTS AND DISCUSSION

This article discusses the application of Problem-Based Learning (PBL) model and Realistic Mathematics Education (RME) approach in mathematics education to improve students' critical thinking skills. The main focus of the research is the integration of the PBL-RME framework with assessment strategies in the context of applying the concept of power dominating sets to solve electricity network optimization problems. This research emphasizes the importance of linking mathematical modeling with real-world situations to equip students with relevant critical and analytical thinking skills.

The first section outlines how the PBL-RME framework is used to engage students in an active and contextualized learning process. Students are encouraged to identify and analyze problems related to power network optimization using the concept of power dominating set. Through activities such as problem analysis, group discussion, and mathematical model development, students learn to integrate theoretical concepts with real problems. The PBL approach encourages students to independently explore problems and develop solution strategies, while the RME approach ensures that learning takes place in a realistic context relevant to everyday life. This process is designed to enhance students' critical thinking skills, including their ability to evaluate solutions, develop effective mathematical models, and devise strategies to optimize the power network.

The second subsection focuses on assessment strategies designed to measure and develop students' critical thinking skills in solving power grid optimization problems. This assessment is designed to evaluate several key aspects of critical thinking, such as fluency of ideas, flexibility in approach, originality of solutions, and students' ability to articulate and explain their problem-solving steps. In the assessment process, students are not only required to produce solutions but also to reflect on their thought processes, refine their approaches based on feedback, and demonstrate a deep understanding of the problem. This strategy fosters learning that is not only outcome-oriented but also process-oriented, providing students with a holistic and meaningful experience.

This third section shows that integrating abstract mathematical concepts, such as power dominating sets, with a problem-based learning approach can significantly improve students' critical thinking skills. By simulating real-world problems, such as power grid optimization, students not only learn the mathematics but also understand its practical relevance. This approach equips them with the skills necessary to solve complex problems in the real world.

In addition, this research makes an important contribution to the development of innovative teaching methods in mathematics education. By connecting mathematical theories with real-world applications through the PBL model and the RME approach, learning becomes more interesting and meaningful, increasing students' engagement while strengthening their understanding of mathematical concepts.

a. 3.1 PBL-RME Learning Framework

The concept of power dominating sets, derived from graph theory, is used in this research to solve power grid optimisation problems, such as determining the minimum number and location of sensors to efficiently monitor the network. Students are invited to understand this abstract mathematical concept while connecting it to practical needs in the real world. In the context of power grids, the vertices in the graph are chosen in such a way as to ensure the dominance of all other vertices efficiently, in order to improve the reliability and efficiency of the grid system.

This approach provides students with opportunities to apply abstract mathematical theories to real problems, thus building a deeper understanding of how mathematics serves as a problem-solving tool in various fields, including energy and technology. The PBL model was chosen for its ability to encourage students to think critically, creatively and collaboratively through direct engagement in complex problem solving. With the RME approach, learning is designed in a real context that is relevant to students' daily lives. This combination not only helps students understand concepts theoretically but also trains them to apply them in situations that require innovative solutions.

The applied learning framework involves six main stages, namely 1) Problem Presentation: Students are introduced to contextual problems related to power network optimization using the concept of power dominating set. 2) Problem Discussion: Students discuss in groups to analyze the problem, identify important facts, and formulate the initial steps of the solution. 3) Group Work: Students work collaboratively to develop problem-solving strategies and design relevant mathematical models. 4) Self-Study: Each student is given the opportunity to explore theories that support problem solving individually. 5) Presentation of Results: Student groups present their solutions, including the mathematical models and optimization strategies that have been designed. 6) Reviewing the Work: Students receive feedback from lecturers and peers, and revise or refine their solutions based on the feedback received.

This research emphasizes the importance of critical thinking skills in mathematics education, which includes the ability to analyze information, design solutions, and evaluate the effectiveness of the approaches used. The problem-based learning process provides a holistic learning experience, where students not only solve problems but also learn to reflect on their steps and consider multiple perspectives to improve the solution.

By presenting real-life relevant problems such as power grid optimization, students become more motivated to learn because they can see first-hand how mathematical concepts are applied to solve real-world challenges. This motivation increases their engagement in the learning process, from the problem analysis stage to the evaluation of results. This learning experience provides students with a deeper understanding and skills that can be applied in the professional world.

This research makes an important contribution in developing an innovative learning model that integrates abstract mathematical theory with practical application. By using PBL framework and RME approach, this model helps bridge the gap between theoretical and practical learning. Students not only understand concepts such as power dominating sets theoretically but are also able to apply them in real contexts.

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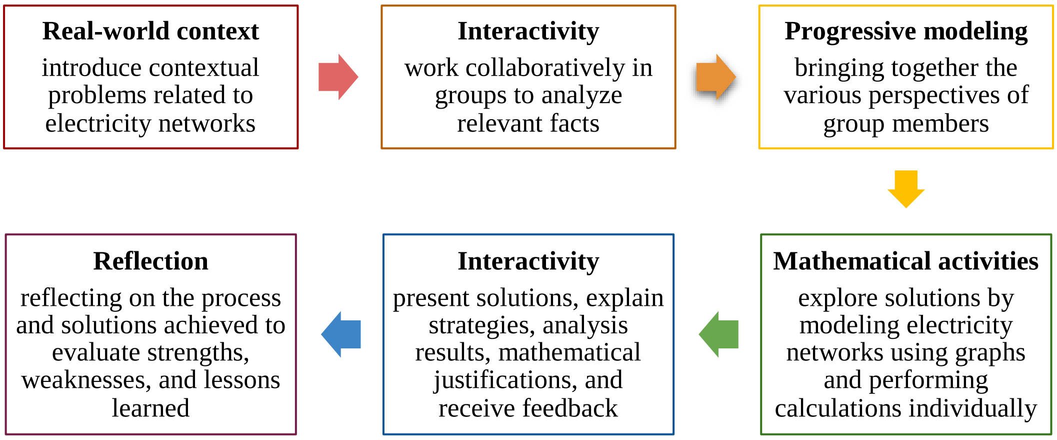

Figure 1. Syntax Framework of PBL Model and RME Approach.

By linking mathematics learning with real-world problems, this research shows how mathematics education can play a role in equipping students with the critical and creative thinking skills needed to face complex challenges. The model is expected to serve as a reference for the development of other learning methods aimed at increasing the relevance, effectiveness and student engagement in mathematics education.

Stage 1: Problem Presentation

The initial stage of learning begins with the presentation of a contextual problem related to the power grid, such as determining the minimum number of sensors needed to monitor the power grid without losing control of the entire system. This problem is presented through scenarios, stories, illustrations, or simulations to help students understand the context. For example, students can see a graph representation of a complex power grid, as in Figure 2. The presentation of the problem aims to attract students' attention and provide challenges that motivate them to explore solutions. At this stage, students read, understand, and identify key elements of the problem, such as the relationship between vertices in the graph and the concept of dominance. This forms the basis for the discussion stage and the development of problem-solving strategies.

Figure 2. Housing Electrical Networks.

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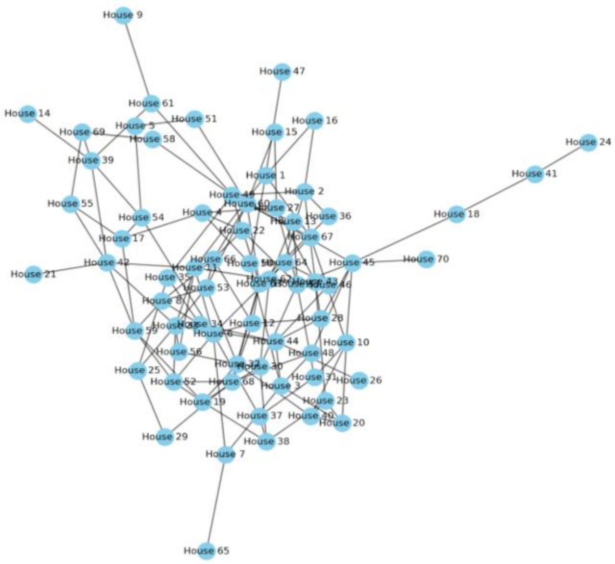

Figure 3. Graph Representation of Housing Network with 70 Vertices.

Stage 2: Problem Discussion

After understanding the problem, students enter the group discussion stage. They work collaboratively to analyse the problem, identify relevant facts, and formulate initial hypotheses. In this discussion, students explore the concept of power dominating set and how this concept is applied to minimise the number of sensors in the electricity network. The group discussion aims to encourage students to think critically and creatively in designing problem-solving strategies. This process also involves peer learning, where students share ideas and strengthen their understanding of relevant concepts. The discussed graph representation of the power grid, as shown in Figure 3, became the basis for developing group solutions.

Stage 3: Return to the Group

After the initial discussion, students return to their groups to bring together the various perspectives of the group members. They evaluate the initial strategies they have devised and refine them based on the input from other members. This stage allows students to integrate information and develop a more mature interim solution. At this stage, students divide tasks effectively among group members to ensure efficiency in problem solving. This process helps build essential collaborative skills, such as teamwork, communication and responsibility management.

Stage 4: Self-Study

The next stage is self-study, where students individually explore and refine their solutions. Students perform further analysis, such as modelling the power grid using graphs, mathematical calculations, and simulation of the solution. They also validate the provisional solutions that have been designed individually. The self-directed learning process aims to hone students' analytical and problem-solving skills. With a focus on individual exploration, this stage provides space for students to ensure that their analytical results are valid and applicable. In addition, students also learn to develop lifelong learning skills, which are crucial in problem-based education.

Stage 5: Results Presentation

Students then present the solutions they have developed to the class. The presentation includes an explanation of the strategy used, the results of the analysis, and a mathematical justification of the solution. In the context of power dominating sets, students explain how the optimal vertices are chosen to dominate the entire graph efficiently. This presentation trains students to organise and convey ideas systematically and receive feedback from lecturers and peers. This process not only develops students' communication skills but also strengthens their understanding through interactive discussions.

Stage 6: Reviewing the Work

The final stage is to review the work. Students and lecturers review the process and the solution, evaluate the effectiveness of the strategies used, and identify the strengths and weaknesses of the solution. In this stage, students reflect on their learning experience to understand the areas that need improvement. This review aims to improve the overall quality of learning, both in terms of concept understanding and students' critical and collaborative thinking skills. This process also provides constructive feedback for future learning.

By applying the PBL model and RME approach, learning the concept of power dominating sets becomes more relevant and meaningful. Students not only learn to understand mathematical theories, but are also able to apply these concepts in solving real problems, such as power network optimization. This learning process improves students' critical, collaborative and analytical thinking skills, which are needed to face real-world challenges.

b. 3.2 Learning Outcomes and Objectives

The purpose of this research is to develop students' conceptual understanding of Power Dominating Set (PDS) in the context of power network optimization. Through this learning, students are expected to understand the basic theory of PDS and its application in solving power network problems. In addition, the learning is also designed to introduce and utilise the Problem-Based Learning (PBL) model and Realistic Mathematics Education (RME) approach as the main strategies in the teaching-learning process.

In the learning process, students will be trained to apply the PBL model and RME approach in analysing, formulating, and solving PDS-based power network optimization problems. This approach is designed not only to improve conceptual understanding, but also to train students' critical thinking skills in dealing with complex problems. Students will be encouraged to dig for information, discuss, and work collaboratively to find innovative and effective solutions.

In addition to the aspects of understanding and application, this study aims to evaluate the extent to which the PBL model and RME approach can improve students' critical thinking skills. These skills are considered important in applied mathematics learning, especially in preparing students to face challenges in the real world. This problem-based learning and contextual approach is also designed to provide a more meaningful learning experience, where mathematical theories are integrated with practical applications.

The results of this study are expected to provide new insights into the effectiveness of implementing innovative learning models in improving students' problem-solving skills, conceptual understanding, and critical thinking ability. In addition, the findings of this study can be the basis for the development of learning methods in the field of applied mathematics, especially those related to power grid optimisation and other complex problems.

c. 3.3 Learning Outcomes and Objectives

Students' critical thinking skills are assessed through four main indicators, namely fluency, flexibility, originality, and elaboration, each of which has sub-indicators as an assessment guide. The first indicator, fluency, measures students' ability to generate various ideas or solutions without hindrance. In the context of power dominating sets, this indicator assesses the extent to which students can identify various approaches to determine the optimal vertices in the electricity network graph. The more ideas that are generated, the more fluent the student's thinking is.

The second indicator is flexibility, which assesses students' ability to switch between different perspectives or approaches in solving problems. In problem-based learning, students are exposed to various electricity network optimisation scenarios, so they must be able to adapt strategies to suit the conditions of the problem. This flexibility is very important in dealing with complex real challenges.

The third indicator, originality, measures the level of creativity and innovation in the solutions developed by students. At this stage, students are encouraged to design original and efficient graph models to dominate the power grid. Unique solutions demonstrate students' ability to think outside the box and generate new ideas relevant to the problem.

The last indicator is elaboration, which measures students' ability to explain, detail and validate their solutions. Elaboration includes students' ability to construct logical arguments, perform detailed mathematical calculations, and reflect on the problem-solving process. This stage not only tests the results but also the students' thinking process in dealing with problems.

The assessment instrument was designed to measure these indicators holistically. Assessment is conducted through contextual problem-based exercises relevant to power grids, such as determining the optimal number of sensors to dominate all nodes in a network. Students are asked to identify the problem, develop a graph model, design a practical solution, and reflect on the approach used. Feedback is given to students to help them refine their solutions and gradually improve their critical thinking skills.

Through the integration of power dominating set in PBL model and Realistic Mathematics Education (RME) approach, this study aims to improve students' critical thinking skills while connecting abstract mathematical concepts with practical applications. This approach is expected to not only strengthen concept understanding but also equip students with relevant skills to face complex challenges in the real world.

Based on Table 1, it is evident that each critical thinking skill indicator evaluates not only the final outcome but also the process students undergo in developing solutions to electricity network optimization problems using the concept of the power dominating set. The fluency indicator focuses on students' ability to generate a variety of ideas and diverse solutions within the context of electricity network problems. This aims to assess the extent to which students can identify multiple alternative solutions without constraints.

Table 1. Indicators of Students' Critical Thinking Skills

Indicator	Sub-Indicator	Test Materials/Assessment Instrument
Basic Clarification	Formulate questions: Evaluate students' ability to formulate critical questions related to power network optimization using power dominating sets.	Instruments: Present an electricity network graph scenario and ask students to formulate two critical questions to understand the network optimization strategy.
	Analyze arguments: Students will be able to evaluate the validity of arguments related to power system optimization strategies.	Instrument: Present an argument, such as "The vertex with the highest degree is always optimal," and ask students to evaluate the correctness of the argument.
Bases for a Decision	Source Credibility: The extent to which students consider the reliability of data or information in making decisions.	Instrument: Provide electricity network data from different sources and ask students to determine the most credible source with justification.
	Observasi dan Analisis: Students' ability to observe graphs and analyze observations to solve problems.	Instrument: Present the graph of the power grid and ask students to identify the dominant vertex based on observation of the graph.
Inference	Deduction: Students' ability to make mathematical deductions based on electrical network data.	Instrument: Provide electricity network data and interest students in making mathematical deductions to determine the dominant node.
	Induction: Students' ability to make generalizations about vertex dominance patterns in power network graphs.	Instrument: Provide electricity network data and interest students in making mathematical deductions to determine the dominant node.
Advanced Clarification	Identify terms: Students will be able to define terms such as "dominant vertex" and relate them to optimization strategies.	Instrument: Students define the term “dominant vertex” in the context of power networks and explain its effect on system optimization.
	Identifying Assumptions: Students' ability to find assumptions that are not explicitly stated in the power grid model.	Instrument: Present the power grid optimization solution and ask students to identify the assumptions used in the model without explicitly stating them.

The flexibility indicator evaluates students' ability to adapt and explore strategies used to solve problems. Students are assessed based on their flexibility in adjusting approaches or strategies as needed, such as determining optimal nodes in an electricity network. This flexibility reflects their critical thinking skills when facing various challenges.

The originality indicator measures the level of creativity and innovation demonstrated by students in designing unique and original solutions. In this context, students are expected to develop efficient and innovative graph models to dominate electricity networks. This indicator aims to assess the extent to which students can produce solutions that deviate from conventional approaches.

The final indicator, elaboration, evaluates students' ability to provide detailed explanations and deeply develop concepts. Students are expected not only to present solutions but also to explain the steps taken, perform mathematical validation, and reflect on the problem-solving process in detail. Elaboration ensures that students understand the relationship between mathematical theory and its practical applications in electricity network optimization.

This assessment framework ensures that students not only grasp the fundamental concepts of the power dominating set but also connect them to real-world applications. This framework integrates abstract mathematical theory with complex real-world contexts, creating a meaningful and relevant learning experience. Through this approach, students are expected to develop profound critical thinking skills, including the ability to analyze, innovate, and effectively explain solutions.

CONCLUSIONS

This study discusses the application of the Problem-Based Learning (PBL) model and the Realistic Mathematics Education (RME) approach in the context of mathematics education, specifically on the topic of the Power Dominating Set, which is used to address electricity network optimization problems. The main objective is to enhance students' critical thinking skills through the implementation of these innovative teaching strategies.

The research findings indicate that 1) the PBL model effectively facilitates students in identifying real-world problems, designing solutions, and evaluating alternative resolutions, directly contributing to the development of critical thinking skills; 2) the RME approach provides a more realistic and relevant learning context for students, enabling them to deeply understand mathematical concepts and connect them to practical applications in the field of electrical networks; 3) the combination of these two methods has proven to have a positive impact on learning outcomes, with students demonstrating significant improvements in analytical abilities, evaluation skills, and solving complex problems.

The implementation of Problem-Based Learning (PBL) and Realistic Mathematics Education (RME) in this research faces several key challenges that need to be addressed to ensure effective learning. One of the main challenges is the complexity of the material. Topics such as the power dominating set and electricity network optimization are abstract and complex, combining mathematical theory with practical applications in engineering. This can be a hurdle, especially for students who do not have a strong foundational understanding. To overcome this, teachers need to design materials that are simple and relevant, using a gradual approach that starts with basic cases before moving on to more complex problems.

In addition, students' limited understanding also poses a significant challenge. Given their diverse backgrounds in mathematics, some students may struggle to grasp the concepts of optimization or electricity networks. A teaching approach that incorporates simple simulations, contextual illustrations, or relatable analogies can help students visualize the concepts and gradually enhance their understanding. Another challenge is the effective integration of PBL and RME. These two models have unique characteristics, requiring careful planning to prevent the learning process from becoming overly complicated. Teachers must be able to connect realistic contexts (the principle of RME) with the stages of problem identification and resolution (the principle of PBL) logically and seamlessly. This involves planning a learning flow that supports students' exploration in understanding real-world problems and solving them with appropriate approaches.

The availability of resources is another hurdle in implementation. Topics like electricity network optimization often require simulation software or specific data that may not always be accessible in schools. To address this limitation, teachers can utilize simple learning tools, such as paper-based simulations or free software relevant to the learning topic. Moreover, PBL and RME-based learning often requires more time than conventional methods, making time constraints a challenge. Teachers need to develop efficient time management plans and prioritize the most important parts of the topic to be covered in each learning session. Another issue is the difficulty in assessing students' critical thinking skills, as this aspect is qualitative and cannot always be measured directly through students' work. Teachers need to use specific assessment rubrics to evaluate critical thinking skills, such as the ability to identify problems, provide logical arguments, and evaluate solutions.

In addition, teachers' readiness is a crucial factor in the successful implementation of PBL and RME. Not all teachers have sufficient experience or training in applying these two models simultaneously, especially for complex topics. Ongoing teacher training is essential to help them understand these approaches and implement them effectively in the classroom. Finally, student engagement often becomes a challenge, especially for students who are not accustomed to active learning methods like PBL and RME. Some students may remain passive or less engaged in discussions and problem exploration. Teachers need to create a supportive learning environment, motivate students, and provide adequate guidance to encourage active participation in the learning process. By addressing these challenges, the implementation of PBL and RME in mathematics education can proceed effectively, helping students improve their understanding of mathematical concepts, critical thinking skills, and the ability to solve real-world problems.

OPEN PROBLEM

This study discusses the application of the Problem-Based Learning (PBL) model and the Realistic Mathematics Education (RME) approach in the context of mathematics education, specifically on the topic of the Power Dominating Set, which is used to address electricity network optimization problems. The main objective is to enhance students' critical thinking skills through the implementation of these innovative teaching strategies.

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