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Forecasting High-Order Multifactor Cross Relations Using Fuzzy Time Series and Frequency Density

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ABSTRACT: The forecasting technique frequently employed for decision-making, initially presented by Song and Cissom utilizing fuzzy logic principles, is Fuzzy Time Series (FTS). The FTS forecasting approach comprises fundamental steps: identifying the conversation universe, segmenting the conversation universe, fuzzification, constructing the fuzzy logic relationship (FLR), and defuzzification. This study involved the modification of the existing methodology to ascertain and categorize the conversational universe, fuzzy logic relationships (FLR), and historical data variations, each utilizing distinct frequency density partitions and cross-relationships, specifically with Indonesian rubber production data. Altering the frequency density partition algorithm to achieve partition universality, followed by fuzzification, cross-relationship analysis, and concluding with the defuzzification procedure. The degree of forecast precision uses the Average Forecasting Error Rate (AFER) to evaluate the revised outcomes against the traditional method. Simulations utilize Indonesian rubber production data from 2000 to 2023, exhibiting an AFER of 3.901%, below 10%, indicating robust forecasting criteria. The forecasting technique frequently employed for decision-making, initially presented by Song and Cissom through the principle of fuzzy logic, is Fuzzy Time Series (FTS). The FTS forecasting approach comprises fundamental steps: identifying the conversation universe, segmenting the conversation universe, fuzzification, constructing the fuzzy logic relationship (FLR), and defuzzification. This study involved the modification of the existing methodology to ascertain and categorize the conversational universe, fuzzy logic relationships, and historical data variations, each employing distinct frequency density partitions and cross-relationships, utilizing Indonesian rubber production data. Altering the frequency density partition algorithm to achieve partition universality, followed by fuzzification, cross-relationship analysis, and concluding with the defuzzification procedure. The degree of forecast precision uses the Average Forecasting Error Rate (AFER) to juxtapose the revised outcomes with the traditional methodology. Simulations utilize Indonesian rubber production data from 2000 to 2023, exhibiting an AFER of 3.901%, below 10%, indicating robust forecasting criteria.

KEYWORDS: Cross-Relationship, Fuzzy Forecasting, Frequency Density Partitioning, High Order, Two Variables.

1. INTRODUCTION

Forecasting is a crucial decision-making instrument that incorporates historical and contemporary data. Numerous traditional techniques are employed to address forecasting issues, including simple moving averages [10][14] *and weighted moving averages* [4], among others. Conventional forecasting methods often suffer from overfitting, where the training data is excessively tailored, leading to diminished accuracy when tested with different datasets. One approach to mitigate overfitting is using Fuzzy Time Series (FTS).

The FTS method is employed to forecast issues where actual data is represented in linguistic terms rather than numerical values, as the information is more substantive [5]. The FTS method has been extensively employed to address forecasting challenges, including stock price prediction [8], rainfall estimation [3], sales forecasting [2], and rubber production forecasting [8]. This strategy assesses uncertainty, enabling more systematic and accountable planning, which many researchers can consistently improve. FTS generally encompasses several stages, such as identifying the universe of discourse, establishing interval duration, and partitioning the universe of discourse. Defining fuzzy sets, fuzzification, constructing fuzzy relations, and defuzzification [13]. With its advancement, several enhancements have occurred in various facets. Initially, ascertain the interval partitioning approach for the universe of discourse. Multiple approaches exist, including interval ratio and frequency density partitioning. The interval ratio algorithm uses fifteen partitions with varying intervals to determine the interval length [15]. Nafis in [8] splits the interval by employing frequency density to reduce superfluous intervals and repartitions the resultant intervals. The results indicate that the frequency density partition method outperforms the interval ratio.

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Secondly, FLR (fuzzy logistic relationship) factors and sequences are applied. The term "factor" refers to a variable that affects forecasting. There are two categories of factors: the primary factor (the element whose anticipated value must be estimated) and the influencing factor (external to the primary factor). FLR forecasting is categorized into two types: one-factor FLR, which considers the primary component alone, and multi-factor FLR, which accounts for the influence of additional factors. FLR is categorized as first-order FLR and high-order FLR according to the order. The FLR employed by [12] first utilized first-order methods, subsequently refined by [6] through high-order techniques applied to the University of Alabama enrollment data, demonstrating superior accuracy compared to first-order methods. The objective of the development is to achieve minimal mistakes.

The researcher aims to develop the anticipation of lower AFER values further than those in prior studies. Accurate and accountable forecasting is essential to address economic management issues effectively, prevent production cost losses, and enhance decision-making processes. Rubber production is a critical sector in the national economy, encompassing agriculture and industry. According to [1] Indonesia is the second largest rubber producer globally for rubber commodities. It possesses significant potential to emerge as a leading producer in international trade. The potential can be optimized by estimating Indonesia's rubber production.

2. MATERIAL AND METHODS

This literature review encompasses national and international publications, books, journals, and prior research pertinent to FTS, multiple factors, high order, cross-relationship (FLR), and frequency density partitions. This study utilizes secondary data from Indonesian rubber plantations, focusing on rubber production as the primary variable and land area as a contributing factor from 2000 to 2023. The data is sourced from BPS (Central Statistics Agency) and is accessible at the following link: https://www.bps.go.id/subject/54/perkebunan.html#subjekViewTab3. The forecasting methodology is outlined as follows:

1. Predicting high-order multiple-factor cross-relationships in FTS based on frequency density. The procedure is as follows:
   a. Identify the scope of discourse (*U*).
   b. Construct a discourse interval universe utilizing a frequency density partition.
   c. Construct a fuzzy set.
   d. Fuzzify historical data by assessing the degree of inclusion inside each fuzzy set.
   e. Construct FLR (HSAFLR, HLAFLR, HSCAFLR, HLCAFLR).
   f. Defuzzification

      Upon completing step 5, the defuzzification of each HSAFLR, HLAFLR, HSCAFLR, and HLCAFLR will be examined.

1. Assess the precision of the forecast by computing the AFER value.
2. Formulate conclusions. This literature review encompasses national and international publications, books, journals, and prior research pertinent to FTS, multifactorial influences, high-order relationships, frequency density partitions, and cross-relationships (FLR). This study utilizes secondary

3. RESULT AND DISCUSSION

FTS analysis of high-order factor cross-relationships utilizing frequency density computed with Microsoft Excel used on Indonesian rubber plantations from 2000 to 2023. The rubber production variable is a dependent variable shown as a time series, identified as the primary factor in this forecast, denoted as X = {1.125,2, 1.723,3, 1.226,6, …, 2.509,3, 2434}. The land area variable is an independent variable within a time series framework, identified as a supporting element in this forecast, denoted as Y^i = {3046, 2838,4, 2825,5, …, 3.263,1, 3.248,8}.

a. We are establishing the scope of discourse (*U*).

   For instance, consider the debate universe U regarding rubber production as the primary variable and the discussion universe U concerning land area as the influencing variable. The discourse universes U and W are defined as follows U = [X_{min} - D_1, X_{max} + D_2] and W= [Y_{min} - D_1, Y_{max} + D_2] where D_1 and D_2 are arbitrary positive values. The minimum rubber production data is 1125.2, and the maximum is 3111.3, with D_1 = 562,6 and D_2 = 1555,65, so the universe of discourse U = [562,6,4666,95] is identical to the universe of discourse W.

b. I am constructing a discourse universe interval by frequency density partitions.

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The subsequent steps in constructing an interval are as follows:

1. Calculating the interval length (l) of the universe of discourse involves subtracting the universe of discourse U and then dividing by m, where m is the number of interval partitions. For instance, we obtain U = [562,6,4666,95] with an interval partition of 9, resulting in an interval length of 456.0389 for rubber production and 114.286 for land area.
2. We are establishing the Interval within the discourse universe. For instance A_1= [562,6,562,6 + 456,0389] = [562,6, 1018,639], A_2 = [1018,639, 1474,678] and this pattern continues until A_9 is obtained.
3. Constructing frequency density involves calculating the frequency for each Interval and splitting the sub-interval based on its frequency density. The initial frequency density (most considerable frequency) is partitioned into four sub-intervals, the subsequent frequency density (second largest frequency) is divided into three sub-intervals, the third frequency density (third largest frequency) is segmented into two sub-intervals, the fourth frequency density (fourth largest frequency) is allocated one sub-interval, and the fifth frequency density (fifth most considerable frequency) is assigned zero sub-intervals (eliminated Interval).

Table 1. Frequency Density

Interval	Frequency	Interval	Frequency
A_1= [562,6, 1018,639]	0	A_8 = [3754,872, 4210,911]	0
A_2 = [1018,639, 1474,678]	3	A_9 = [4210,911, 4666,95]	0
A_3 = [1474,678, 1930,717]	4		
A_4 = [1930,717, 2386,756]	5		
A_5 = [2386,756, 2842,794]	8		
A_6 = [2842,794, 3298,833]	4		
A_7 = [3298,833, 3754,872]	0		

Table I indicates that the initial frequency density is A_5 divided into four additional subintervals. The second frequency density A_4 is subdivided into three new subintervals. The third frequency density is A_3, A_6 the division into two new subintervals. The fourth frequency density is A_2, partitioned into one new subinterval. An illustration to derive four new sub-intervals from. An illustration to derive four new sub-intervals from A_5 is (2842,794-2386,756)/4 = 114,0097. Consequently, the new intervals are [2386,756, 2386,756+114,0097] = [2386,756, 2500,765], [2500,765, 2614,775], [2614,775, 2728,785], [2728,785, 2842,794]. Identical to that of the others.

Table 2. New Intervals

Interval	Frequency	New Intervals	Frequency
A_1= [562,6, 1018,639]	0	A_1= [1018,639, 1474,678]	3
A_2=[1018,639, 1474,678]	3	A_2= [1474,678, 1702,697]	1
A_3=[1474,678, 1930,717]	4	A_3= [1702,697, 1930,717]	3
A_4=[1930,717, 2386,756]	5	A_4= [1930,717, 2082,73]	1
A_5=[2386,756, 2842,794]	8	A_5= [2082,73, 2234,743]	3
A_6=[2842,794, 3298,833]	4	A_6= [2234,743, 2386,756]	1
A_7=[3298,833, 3754,872]	0	A_7= [2386,756, 2500,765]	2
A_8=[3754,872, 4210,911]	0	A_8= [2500,765, 2614,775]	4
A_9 = [4210,911, 4666,95]	0	A_9= [2614,775, 2728,785]	1
		A_{10}=[2728,785,2842,794]	1
		A_{11}=[2842,794,3070,814]	3
		A_{12}=[3070,814,3298,833]	1

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c. Building a fuzzy set.

Establishing a fuzzy set $\{A_l \mid l = 1, 2, 3, \ldots, k+1\}$ pada $\{U_j \mid j = 1, 2, 3, \ldots, k\}$ by Definition II.5 and Definition II.6 as outlined:

A_1 = [1018,639, 1018,639, 1474,678] with the corresponding membership function:

$$\mu_a(x) = \begin{cases} \frac{1474{,}678 - x}{1474{,}678 - 1018{,}639} & ,1018{,}639 \le x \le 1474{,}678 \\ 0 & ,others \end{cases}$$

A_2 = [1474,678-456,0389, 1474,678, 1474,678+456,0389]=[1018,639, 1474,678, 1702,697] with membership function:

$$\mu_a(x) = \begin{cases} \frac{x - 1018{,}639}{1474{,}678 - 1018{,}639} & ,1018{,}639 \le x \le 1474{,}678 \\ \frac{1702{,}697 - x}{1702{,}697 - 1474{,}678} & ,1474{,}678 \le x \le 1702{,}697 \\ 0 & ,others \end{cases}$$

A_3 = [1702,697-228,0194, 1702,697, 1702,697+228,0194]=[1474,678, 1702,697, 1930,717] with membership function:

$$\mu_a(x) = \begin{cases} \frac{x - 1474{,}678}{1702{,}697 - 1474{,}678} & ,1474{,}678 \le x \le 1702{,}697 \\ \frac{1930{,}717 - x}{1930{,}717 - 1702{,}697} & ,1702{,}697 \le x \le 1930{,}717 \\ 0 & ,others \end{cases}$$

Subsequently, the membership value of each fuzzy set is determined using fuzzy intervals and triangular membership functions corresponding to the u_i rule.

The precise value of 1125,2 is located in language A_1 with a membership value of 0,766333

$$\mu_a(1125{,}2) = \begin{cases} \frac{1474{,}678 - 1125{,}2}{1474{,}678 - 1018{,}639} & ,1018{,}639 \le x \le 1474{,}678 \\ 0 & ,others \end{cases}$$

$$\mu_a(1125{,}2) = \begin{cases} 0{,}766333 & ,1018{,}639 \le x \le 1474{,}678 \\ 0 & ,others \end{cases}$$

The precise value of1125,2 resides in linguistic A_2 with a membership value of 0,233667

$$\mu_a(1125{,}2) = \begin{cases} \frac{1125{,}2 - 1018{,}639}{1474{,}678 - 1018{,}639} & ,1018{,}639 \le x \le 1474{,}678 \\ 0 & ,1235{,}389 \le x \le 1345{,}78 \\ 0 & ,others \end{cases}$$

$$\mu_a(1125{,}2) = \begin{cases} 0{,}233667 & ,1018{,}639 \le x \le 1474{,}678 \\ 0 & ,1474{,}678 \le x \le 1702{,}697 \\ 0 & ,others \end{cases}$$

The precise value of 1125,2 corresponds to linguistic categories A_1 and A_2, with membership values of 0,766333 and 0,233557, respectively. The maximum membership value of 0,766333 is associated with linguistics A_1 indicating that in the 2000 data, fuzzification is performed by A_1. Additional membership values and fuzzification are pursued using the same analogy as previously mentioned.

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Table 3. Fuzzy Triangle Set

Year	Interval Fuzzy	Fuzzy	Year	Interval Fuzzy	Fuzzy
2000	[1018,639, 1018,639, 1474,678]	A_1	2012	[3070,814, 3298,833, 3298,833]	A_7
2001	[1018,639, 1474,678, 1702,697]	A_3	2013		A_9
2002	[1474,678, 1702,697, 1930,717]	A_1	2014		A_9
2003	[1702,697, 1930,717, 2082,73]	A_2	2015		A_9
2004	[1930,717, 2082,73, 2234,743]	A_3	2016		A_{10}
2005	[2082,73, 2234,743, 2386,756]	A_4	2017		A_{12}
2006	[2234,743, 2386,756, 2500,765]	A_5	2018		A_{12}
2007	[2386,756, 2500,765, 2614,775]	A_6	2019		A_{11}
2008	[2500,765, 2614,775, 2728,785]	A_5	2020		A_8
2009	[2614,775, 2728,785, 2842,794]	A_4	2021		A_{11}
2010	[2728,785, 2842,794, 3070,814]	A_6	2022		A_8
2011	[2842,794, 3070,814, 3298,833]	A_7	2023		A_7

d. Fuzzification of historical data by assessing the degree of inclusion inside each fuzzy collection.

Table 4. Fuzification

Year	Rubber Production Fuzzyfikasi	Fuzzyfikasi Land Area	Year	Rubber Production Fuzzyfikasi	Fuzzyfikasi Land Area
2000	A_1	B_4	2012	A_7	B_4
2001	A_3	B_3	2013	A_9	B_4
2002	A_1	B_3	2014	A_9	B_5
2003	A_2	B_3	2015	A_9	B_5
2004	A_3	B_3	2016	A_{10}	B_5
2005	A_4	B_3	2017	A_{12}	B_5
2006	A_5	B_3	2018	A_{12}	B_6
2007	A_6	B_4	2019	A_{11}	B_6
2008	A_5	B_4	2020	A_8	B_6
2009	A_4	B_4	2021	A_{11}	B_7
2010	A_6	B_4	2022	A_8	B_6
2011	A_7	B_4	2023	A_7	B_6

e. Constructing FLR.

Determine the relationship among the four FLRs: HSAFLR, HLAFLR, HSCAFLR, and HLCAFLR.
The relationship of HSAFLR present in S is as follows:

Let A_{f_u} denote the fuzzification of the principal factor observation at the time A_{f_u} ($u = 1,2,\ldots,h$). N is the quantity of accessible HSAFLR identified in S, based on the count of N forecasts $x^*_{t_s}$ at time t, the calculation can be derived from two scenarios ($f_u, l_r \in$ {1,2,…,k+1}, $u = 1,2,\ldots,h, h > 1, r = 1,2,\ldots,N$) :
Scenario 1 for N ≠ 0

Considering a quantity of N FLRs with:

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Table 5. Yearly Transition Patterns in HSAFLR

Year	HSAFLR	Year	HSAFLR
2000		2012	$A_4, A_6, A_7 \rightarrow A_7$
2001		2013	$A_6, A_7, A_7 \rightarrow A_9$
2002		2014	$A_7, A_7, A_9 \rightarrow A_9$
2003	$A_1, A_3, A_1 \rightarrow A_2$	2015	$A_7, A_9, A_9 \rightarrow A_9$
2004	$A_3, A_1, A_2 \rightarrow A_3$	2016	$A_9, A_9, A_9 \rightarrow A_{10}$
2005	$A_1, A_2, A_3 \rightarrow A_4$	2017	$A_9, A_9, A_{10} \rightarrow A_{12}$
2006	$A_2, A_3, A_4 \rightarrow A_5$	2018	$A_9, A_{10}, A_{12} \rightarrow A_{12}$
2007	$A_3, A_4, A_5 \rightarrow A_6$	2019	$A_{10}, A_{12}, A_{12} \rightarrow A_{11}$
2008	$A_4, A_5, A_6 \rightarrow A_5$	2020	$A_{12}, A_{12}, A_{11} \rightarrow A_8$
2009	$A_5, A_6, A_5 \rightarrow A_4$	2021	$A_{12}, A_{11}, A_8 \rightarrow A_{11}$
2010	$A_6, A_5, A_4 \rightarrow A_6$	2022	$A_{11}, A_8, A_{11} \rightarrow A_8$
2011	$A_5, A_4, A_6 \rightarrow A_7$	2023	$A_8, A_{11}, A_8 \rightarrow A_7$

Secondly, the HLAFLR relation present in L is as follows:

Let $A_{f_{p_u}}$ represent the fuzzification of the primary factor observations at the time (t-p_u). According to Definition II.14, HLAFLR the accessible HLAFLR will be ascertained using the fuzzy set $A_{f_{p_u}}$ $(u = 1,2,\dots,h)$. N is the number of accessible HSAFLRs identified in L, corresponding to the number N of forecasts $x_{t_l}^*$. At time t, calculations can be derived from two scenarios. (p_u, $p_h \in Z^+$ $p_{(u+1)} > p_u, p_h > h, f_{p_u}, f_{p_h}, l_r \in$ {1,2,…,k+1}, $u = 1,2,\dots,(h-1), h > 1, r = 1,2,\dots,N$) :

Case 1 for N ≠ 0

Considering a quantity of N FLRs with:

Table 6. Annual Transition Sequences in HLAFLR (2000–2023)

Year	HLAFLR	Year	HLAFLR
2000		2012	A_5,$A_4, A_6, \rightarrow A_7$
2001		2013	$A_4, A_6, A_7 \rightarrow A_9$
2002		2014	$A_6, A_7, A_7 \rightarrow A_9$
2003	NA, $A_1, A_3 \rightarrow A_2$	2015	$A_7, A_7, A_9 \rightarrow A_9$
2004	$A_1, A_3, A_1 \rightarrow A_3$	2016	$A_7, A_9, A_9 \rightarrow A_{10}$
2005	$A_3, A_1, A_2 \rightarrow A_4$	2017	$A_9, A_9, A_9 \rightarrow A_{12}$
2006	$A_1, A_2, A_3 \rightarrow A_5$	2018	$A_9, A_9, A_{10} \rightarrow A_{12}$
2007	A_2,$A_3, A_4 \rightarrow A_6$	2019	$A_9, A_{10}, A_{12} \rightarrow A_{11}$
2008	$A_3, A_4, A_5 \rightarrow A_5$	2020	$A_{10}, A_{12}, A_{12} \rightarrow A_8$
2009	$A_4, A_5, A_6 \rightarrow A_4$	2021	$A_{12}, A_{12}, A_{11} \rightarrow A_{11}$
2010	$A_5, A_6, A_5 \rightarrow A_6$	2022	$A_{12}, A_{11}, A_8 \rightarrow A_8$
2011	$A_6, A_5, A_4 \rightarrow A_7$	2023	$A_{11}, A_8, A_{11} \rightarrow A_7$

Third, the HSCAFLR relation present in $SC_i(i = 1,2,\dots,n)$ is as follows:

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Let $B_{j_u}^i$ denote the fuzzification of the observation of the influence factor $(i+1)$ at the time (t- u). According to Definition II.15, the accessible HSCAFLR will be ascertained using the fuzzy set $B_{j_u}^i$ ($u = 1,2,\dots,h$). N represents the quantity of HSCAFLR present in SC_i, corresponding to the number N of predictions $x_{tSc_i}^*$ At time t, the calculation can be derived from two scenarios (j_u ∈ {1,2,…, k_i+1}, l_r∈ {1,2,…,k+1}, u=1,2,…,h, h>1, i=1,2,…,n, r=1,2,…, N):
Scenario 1 for N ≠ 0
Considering a quantity of N FLRs with:

Table 7. HSCAFLR Category: Annual Transitions and Changes

Tahun	HSCAFLR	Tahun	HSCAFLR
2000		2012	$B_3, B_4, B_4 \rightarrow A_9$
2001		2013	$B_4, B_4, B_4 \rightarrow A_9$
2002		2014	$B_4, B_4, B_4 \rightarrow A_{10}$
2003	$B_4, B_3, B_3 \rightarrow A_2$	2015	$B_4, B_4, B_4 \rightarrow A_{12}$
2004	$B_3, B_3, B_2 \rightarrow A_3$	2016	$B_4, B_4, B_4 \rightarrow A_{12}$
2005	$B_3, B_2, B_2 \rightarrow A_4$	2017	$B_4, B_4, B_5 \rightarrow A_{11}$
2006	$B_2, B_2, B_2 \rightarrow A_5$	2018	$B_4, B_5, B_6 \rightarrow A_8$
2007	$B_2, B_2, B_3 \rightarrow A_6$	2019	$B_5, B_6, B_6 \rightarrow A_{11}$
2008	$B_2, B_3, B_3 \rightarrow A_5$	2020	$B_6, B_6, B_7 \rightarrow A_8$
2009	$B_3, B_3, B_3 \rightarrow A_4$	2021	$B_6, B_7, B_6 \rightarrow A_7$
2010	$B_3, B_3, B_3 \rightarrow A_6$	2022	$B_3, B_4, B_4 \rightarrow A_9$
2011	$B_3, B_3, B_3 \rightarrow A_7$	2023	$B_4, B_4, B_4 \rightarrow A_9$

Fourth, the HLCAFLR relation that is accessible in 〖LC〗_i (i=1,2,…,n) is summarized as follows:

Let $B_{j_{p_u}}^i$ represent the fuzzification of the observation of the influencing factor $(i+1)$ at the time (t-p_u). According to Definition II.16, the accessible HLCAFLR will be ascertained using the fuzzy set $B_{j_{p_u}}^i$($u = 1,2,\dots,h$). N represents the quantity of HLCAFLR present in LC_i, The number N of forecasts $x_{tLC_i}^*$ at time t may be computed based on two scenarios (j_{p_u} ∈ {1,2,…, k_i+1}, l_r ∈ {1,2,…,k+1}, $u = 1,2,\dots,h, h > 1, i = 1,2,\dots,n, r = 1,2,\dots,N$) :
Case 1 for N ≠ 0
Considering a quantity of N FLRs with :

Table 8. HLCAFLR Evolution By Year (2000-2023)

Year	HLCAFLR	Year	HLCAFLR
2000		2012	$B_3, B_3, B_4 \rightarrow A_9$
2001		2013	$B_3, B_4, B_4 \rightarrow A_9$
2002		2014	$B_4, B_4, B_4 \rightarrow A_{10}$
2003	NA, $B_4, B_3 \rightarrow A_2$	2015	$B_4, B_4, B_4 \rightarrow A_{12}$
2004	$B_4, B_3, B_3 \rightarrow A_3$	2016	$B_4, B_4, B_4 \rightarrow A_{12}$
2005	$B_3, B_3, B_2 \rightarrow A_4$	2017	$B_4, B_4, B_4 \rightarrow A_{11}$
2006	$B_3, B_2, B_2 \rightarrow A_5$	2018	$B_4, B_4, B_5 \rightarrow A_8$
2007	$B_2, B_2, B_2 \rightarrow A_6$	2019	$B_4, B_5, B_6 \rightarrow A_{11}$
2008	$B_2, B_2, B_3 \rightarrow A_5$	2020	$B_5, B_6, B_6 \rightarrow A_8$
2009	$B_2, B_3, B_3 \rightarrow A_4$	2021	$B_6, B_6, B_7 \rightarrow A_7$

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2010	$B_3, B_3, B_3 \rightarrow A_6$	2022	$B_3, B_3, B_4 \rightarrow A_9$
2011	$B_3, B_3, B_3 \rightarrow A_7$	2023	$B_3, B_4, B_4 \rightarrow A_9$

f. Defuzzification.

Determine the predicted value for each FLR, specifically $\{x^*_{t_s}, x^*_{t_l}, x^*_{t_{SC_i}}, x^*_{t_{LC_i}}\}$ from which the ultimate forecast value x^* can be derived. The subsequent is the comprehensive forecasting procedure :

Forecasting for 2003 (x^*_{2003}) is conducted by computing $x^*_{2003_S}$, $x^*_{2003_L}$, $x^*_{2003_{SC}}$, $x^*_{2003_{LC}}$ as follows:

Compute the projected value $x^*_{2003_S}$ based on the accessible HSAFLR for S. The data for 2000, 2001, and 2002 (x_{2000}, x_{2001}, x_{2002}) from the initial factor is ambiguous due to A_1, A_5 By searching for S, we may identify the available HSAFLR A_1, A_3, A_1 as follows :

$A_1, A_3, A_1 \rightarrow A_2$

If N ≠ 0, then apply case 1. Utilizing the aforementioned FLR, we can get the forecast $x^*_{2002_S}$ as follows:

$$x^*_{t_s} = \frac{v_1\, x\,(l_1) + v_2\, x\,(l_2) + \cdots + v_N\, x\,(l_N)}{v_1 + v_2 + \cdots + v_N}$$

$$x^*_{2003_S} = \frac{1\, x\,(2)}{1} = 1189{,}65$$

Where $x(3) = \frac{0{,}5\, m(3-1) + Q(3) + 0{,}5m(3+1)}{2}$

$= \frac{0{,}5\, m(2) + Q(3) + 0{,}5m(4)}{2}$

$= 1189{,}65$

Thus, we obtain:

$$x^*_{2003_S} = \frac{1\, x\, 1180}{1} = 1189{,}65$$

Subsequently, compute the forecast $x^*_{2003_L}$ from HLAFLR based on the available L. The data for 2000, 2001, 2002 (x_{1999}, x_{2000}, x_{2001}) from the initial factor is obscured by NA, A_1, and A_3 By searching on L, one can locate the HLAFLR for data with premises NA, A_1, and A_3, which are as follows:

NA, A_1, $A_3 \rightarrow A_2$

If N ≠ 0, then apply case 1. Utilizing the aforementioned FLR, we can get the forecast $x^*_{2003_S}$ as follows :

$$x^*_{t_s} = \frac{v_1\, x\,(l_1) + v_2\, x\,(l_2) + \cdots + v_N\, x\,(l_N)}{v_1 + v_2 + \cdots + v_N}$$

$$x^*_{2003_L} = \frac{1\, x\,(3)}{1} = 1189{,}65$$

Where $x(3) = \frac{0{,}5\, m(3-1) + Q(3) + 0{,}5m(3+1)}{2}$

$= \frac{0{,}5\, m(2) + Q(3) + 0{,}5m(4)}{2}$

$= 1189{,}65$

Later, compute the forecast $x^*_{2003_{SC}}$ from the HSCAFLR using the available SC_i. The data for the years 2000, 2001, 2002 (x_{2000}, x_{2001}, x_{2002}) of the primary factor, along with the time series of the influencing factors, are B_4, B_3, and B_3 respectively. The HSCAFLR identified at the available SC_i for the premises B_4, B_3, and B_3 is as follows:

$B_4, B_3, B_3 \rightarrow A_2$

Using the FLR above, the forecast $x^*_{2003_{SC}}$ can be obtained as follows :

$$x^*_{2003_{SC}} = \frac{1\, x\,(3)}{1} = 1189{,}65$$

Where $x(3) = \frac{0{,}5\, m(3-1) + Q(3) + 0{,}5m(3+1)}{2}$

$= \frac{0{,}5\, m(2) + Q(3) + 0{,}5m(4)}{2}$

$= 1189{,}65$

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Then, using the available LC_i, compute the forecast $x^*_{2003_{LC}}$ from HLCAFLR. The data for 1999, 2000, and 2001 (x_{1999}, x_{2000}, x_{2001}) from the primary factor and the time series of the influencing factors are NA, B_4, and B_3 respectively. The HLCAFLR identified on the accessible LC_i for the locations NA, B_4, B_3 is as follows:

NA, $B_4, B_3 \rightarrow A_2$

Utilizing the aforementioned FLR, the forecast $x^*_{2003_{LC}}$ can be derived as follows:

$$x^*_{2003_{LC}} = \frac{1\,x\,(3)}{1} = 1189{,}65$$

Where $$x(3) = \frac{0{,}5\,m(3-1)+Q(3)+0{,}5m(3+1)}{2} = \frac{0{,}5\,m(2)+Q(3)+0{,}5m(4)}{2} = 1189{,}65$$

The conclusive predicted outcome for 2003 is achieved, namely

$$2003^* = \frac{x^*_{2003_S}+x^*_{2003_L}+x^*_{2003_{SC}}+x^*_{2003_{LC}}}{4} = \frac{1189{,}65+1189{,}65+1189{,}65+1189{,}65}{4} = 1189{,}65.$$

The analogous observation for the subsequent year. The comprehensive forecast values are presented in the subsequent table:

Table 9. Conclusive Forecast Outcomes

Year	Final Forecast Results	Year	Year
2000	NA	2012	2301,248
2001	NA	2013	2557,77
2002	NA	2014	2557,77
2003	1189,653	2015	2557,77
2004	1588,688	2016	2671,78
2005	1797,705	2017	2956,804
2006	2006,723	2018	2956,804
2007	2158,736	2019	2814,292
2008	2006,723	2020	2443,76
2009	1797,705	2021	2814,292
2010	2158,736	2022	2443,76
2011	2301,248	2023	2301,248

This study uses AFER for error evaluation, adhering to the established AFER standards.

$$AFER = \frac{\sum_{t=1}^{n} \frac{|A_t - F_t|}{A_t}}{n} \times 100\%$$

The subsequent table displays the comprehensive computations:

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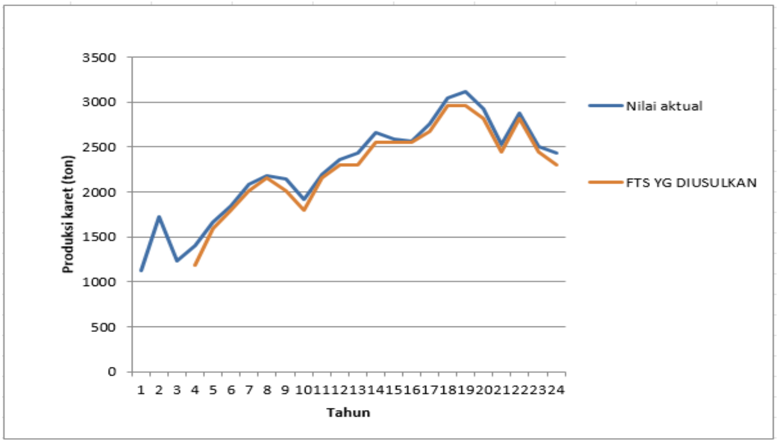
Table 10. AFER Computation

Year	Actual Value (A_t)	$\frac{\lvert A_t - F_t \rvert}{A_t}$	Year	Actual Value (A_t)	$\frac{\lvert A_t - F_t \rvert}{A_t}$
2000	1125,2		2012	2429,5	0,052789354
2001	1723,3		2013	2655,94	0,036962379
2002	1226,6		2014	2583,4	0,009920981
2003	1396,2	0,147934771	2015	2568,6	0,004216251
2004	1662	0,044111011	2016	2754,7	0,030101332
2005	1838,7	0,022295467	2017	3050,2	0,030619577
2006	2082,6	0,036433714	2018	3111,3	0,04965636
2007	2176,7	0,008252809	2019	2926,6	0,038374901
2008	2148,7	0,066075698	2020	2533,5	0,035421189
2009	1918	0,062718809	2021	2877,9	0,022102222
2010	2193,4	0,015803724	2022	2509,3	0,026118672
2011	2359,8	0,02481216	2023	2434	0,054540565
				AFER	0,039012474

Table 10 indicates that the graph of the high-order multi-factor FTS forecasting method, employing cross-relationships derived from frequency density divisions, closely aligns with the actual value, as demonstrated by the accompanying visual representation:



The figure presents a comparison graph illustrating the actual values vs the forecast values of the high-order multi-factor FTS cross-relationship based on frequency density as shown in Figure 1.

4. CONCLUSION

The AFER value calculated for the high-order multi-factor FTS model, incorporating cross-relationships based on frequency density, was 3.901%, less than 10%. According to the AFER criterion, the forecasting demonstrates excellent standards for predicting Indonesian rubber production.

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