



## Application of Automatic Clustering and Fuzzy Time Series Type-2 in Indonesia Composite Index

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**ABSTRACT:** Forecasting the Indonesia Composite Index (ICI) is one of the important efforts in making investment decisions in the capital market. This index helps investors see changes in stock prices directly, making it easier to know whether stock prices are rising or falling. In this research, a combined approach between Fuzzy Time Series (FTS) Type-2 and an automatic clustering algorithm is applied to improve the accuracy of ICI forecasting. The advantage of FTS Type-2 is that it can provide more accurate results than FTS Type-1 because it can express more information. At the same time, automatic clustering is also used because it can partition the universe of discourse efficiently. The data used is monthly data on the closing, highest, and lowest prices of the Indonesia Composite Index from January 2019 to July 2024. The results of this study show an AFER value of 0.6827 and a forecasting accuracy of 99.317%. This shows that the forecasting results using FTS Type-2 and automatic clustering algorithms provide excellent forecasting results. This research has the novelty of using automatic clustering in determining the interval of Fuzzy Time Series Type-2 for ICI forecasting.

**KEYWORDS:** Automatic Clustering, AFER, Fuzzy Time Series Type-2, Indonesia Composite Index.

### INTRODUCTION

The capital market has a crucial role in Indonesia. The capital market is a place where governments and companies raise funds from the public to finance various needs, such as business expansion or project development. The capital market brings together those who need funds and those who want to invest for profit. As the capital market develops and more people become familiar with it, information about the capital market becomes very important. The stock price index provides information and an overview of the share price movements of a particular company. Understanding the movement of the stock price index in the stock market is important because we can see a picture of past stock market conditions. The Indonesia Stock Exchange uses a stock price index in the form of the Indonesia Composite Index. In capital market activities, the ICI has a major influence on investors' decisions to interact. This index helps investors see changes in stock prices directly, so they can easily tell whether stock prices are rising or falling [1]. Therefore, forecasting the Indonesia Composite Index is necessary to support investment decisions.

One approach that can be used to forecast financial data such as the Indonesia Composite Index is Fuzzy Time Series (FTS). The FTS model was introduced by [2] which is a time series forecasting model based on fuzzy set theory. This method is famous for being easy to use and very flexible in various situations [3]. Some studies related to FTS include [4], [5], [6], [7]. It allows us to model uncertain and fluctuating data better than conventional methods. Along with the increasing data complexity and uncertainty, the Type-2 Fuzzy Time Series method emerged as an improvement of the Type-1 FTS method. Type-2 FTS uses more than one variable, and their membership degrees are considered as fuzzy sets. The advantage of Type-2 FTS lies in its ability to provide more accurate results than Type-1 FTS because it can express more information [8]. However, one of the challenges in using Fuzzy Time Series methods is data partitioning, which is determining the best way to divide data into intervals or groups. The length of this interval will have an impact on the difference in forecasting results obtained [9]. This is where the automatic clustering algorithm plays an important role. The automatic clustering algorithm is a method to group a number of data into several clusters or groups. Tirta's research uses automatic clustering to obtain several intervals which are then used in the application of fuzzy logic to predict the number of sales [10]. Automatic clustering algorithms excel at efficiently dividing data sets into optimal intervals, as they can determine the number of groups automatically [11]. In addition, the automatic clustering algorithm is also used to determine the number of clusters needed, as shown in a recent study by [12].



Based on this explanation, this research will apply the Fuzzy Time Series Type-2 model to forecast the Indonesia Composite Index with the aim of helping investors determine the best time to buy or sell the ICI in order to gain profit. To determine the interval length, this research will use the automatic clustering algorithm.

**RESEARCH METHODOLOGY**

**Sources of Data and Research Variables**

This study uses secondary data containing information on closing prices, highest price, and lowest price of monthly data of the Indonesia Composite Index from 2019 - 2024. To train the Type-1 FTS model, 67 ICI closing price data from January 2019 to July 2024 were used. Meanwhile, for testing the Type-2 FTS model, the lowest, highest, and closing price data of ICI from January 2023 - July 2024 were used. All data was obtained from the website <https://finance.yahoo.com> with the code JKSE.

**Methods of Analysis**

The FTS model used in this paper is the Chen model because this model provides simpler calculations and produces more accurate forecasts [13]. The process of forecasting with Type-2 FTS using an automatic clustering algorithm starts with forecasting using Type-1 FTS as follows:

- 1. Establishing a universe of speech from pre-existing data. [14]:

$$U = [D_{min} - D_1, D_{max} + D_2]$$

where

$D_{min}$  : the smallest data

$D_{max}$  : the biggest data

$D_1$  : the first positive number

$D_2$  : a second positive number

- 2. Determine the length and number of intervals using automatic clustering on the closing price variable.

An automatic clustering method is an algorithm that serves to group data into specific groups (clusters) and then convert them into intervals of varying length for each interval. [10]. Here are the steps in the automatic clustering algorithm:

**Step 1:** Organize numerical data in order from smallest to largest, and assume that no two data are the same.

$$d_1, d_2, d_3, \dots, d_i, \dots, d_n$$

Then calculate *average\_diff* or the average of the difference between 2 consecutive numeric data  $d_1, d_2, \dots, d_n$ , with the formula:

$$average\_diff = \frac{\sum_{i=1}^{n-1} (d_{i+1} - d_i)}{n - 1}$$

**Step 2:** Take the first numerical data which is the smallest data then place it in the current cluster or create a new cluster according to the principles:

Principle 1: Let's assume that the current cluster is the first cluster and has only one data or  $d_1$  and assume  $d_2$  is the data close to  $d_1$ :

$$\{d_1\}, d_2, d_3, \dots, d_i, \dots, d_n$$

if  $d_2 - d_1 \leq average\_diff$ , then add  $d_2$  to the existing cluster with member  $d_1$ . If not possible, create a new cluster containing  $d_2$ .

Principle 2: Assume the current cluster is not the first cluster, and  $d_j$  is the only data in the cluster. Suppose  $d_k$  is the closest data to  $d_j$ , while  $d_i$  is the data with the largest value in the previous cluster.

$$\{d_1\}, \dots, \{\dots, d_i\}, \{d_j\}, d_k, \dots, d_n$$

if  $d_k - d_j \leq average\_diff$  and  $d_k - d_j \leq d_j - d_i$ , then put  $d_k$  into the cluster that  $d_j$  belongs to, otherwise create a new cluster that has  $d_k$  members.



Principle 3: Misalkan cluster saat ini bukanlah cluster pertama, dan  $d_i$  merupakan data terbesar di dalam cluster tersebut. Assume that  $d_j$  is the closest data to  $d_i$ .

$$\{d_1\}, \dots, \{\dots\}, \{\dots, d_i\}, d_j, \dots, d_n$$

if  $d_j - d_i \leq average\_diff$  and  $d_j - d_i \leq cluster\_diff$ , then put  $d_j$  into the cluster that contains  $d_i$ . Otherwise, create a new cluster for  $d_j$ . Calculate  $cluster\_diff$  using the following formula:

$$cluster\_diff = \frac{\sum_{i=1}^{n-1} (c_{i+1} - c_i)}{n - 1}$$

where  $cluster\_diff$  is the average of the current cluster and  $c_1, c_2, \dots, c_n$  are the data in the current cluster.

**Step 3:** Based on the clarification results of step 2, then adjust the cluster contents based on the principles:

Principle 1: If there are more than two data in the cluster, keep the smallest and largest data, then delete the other data.

Principle 2: If the cluster has two data, then keep all of them.

Principle 3: If the cluster has only one  $d_q$  data, then add data with the value of  $d_q - average\_diff$  and  $d_q + average\_diff$  to the cluster according to the situation:

Situation 1: If the first cluster, then remove  $d_q - average\_diff$  and keep  $d_q$ .

Situation 2: If the last cluster, then remove  $d_q + average\_diff$  and keep  $d_q$ .

Situation 3: If  $d_q - average\_diff$  is smaller than the smallest data value in the previous cluster, then principle 3 is not applied.

**Step 4:** Assume the result of step 3 as follows:

$$\{d_1, d_2\}, \{d_3, d_4\}, \{d_5, d_6\}, \dots, \{d_r\}, \{d_s, d_t\}, \dots, \{d_{n-1}, d_n\}$$

Transform the cluster results into adjacent clusters through sub-steps:

1.1 Convert the first cluster  $\{d_1, d_2\}$  into interval  $[d_1, d_2]$ .

1.2 If the current interval is  $[d_i, d_j]$  and the current cluster is  $\{d_k, d_1\}$ , then:

1.2.1 If  $d_j \geq d_k$ , then form an interval  $[d_j, d_1]$ . The interval  $[d_j, d_1]$  now becomes the current interval and the next cluster  $\{d_m, d_n\}$  becomes the current cluster.

1.2.2 If  $d_j < d_k$ , then change the current cluster  $\{d_k, d_1\}$  to the interval  $[d_k, d_1]$  and create a new interval  $[d_j, d_k]$  from the intervals  $[d_i, d_j]$  and  $[d_k, d_1]$ . Now  $[d_k, d_1]$  becomes the current interval and the next cluster  $\{d_m, d_n\}$  becomes the current cluster. If now the current interval is  $[d_i, d_j]$  and the current cluster is  $\{d_k\}$ , then change the current interval  $[d_i, d_j]$  to  $[d_i, d_k]$ . Now  $[d_i, d_k]$  is the current interval and the next interval becomes the current interval.

1.3 Repeat sub-steps 1.1 and 1.2 until all clusters become intervals.

**Step 5:** The result of step 4 divides the interval into  $p$  sub-intervals, where  $p \geq 1$ .

3. Define Type-1 fuzzy set.

$$\begin{aligned} A_1 &= 1/u_1 + 0,5/u_2 + 0/u_3 + 0/u_4 + \dots + 0/u_n \\ A_2 &= 0,5/u_1 + 1/u_2 + 0,5/u_3 + 0/u_4 + \dots + 0/u_n \\ &\vdots \\ A_k &= 0/u_1 + 0/u_2 + 0/u_3 + \dots + 0,5/u_{n-1} + 0/u_n \end{aligned}$$

4. Determine Fuzzy Logical Relationship (FLR) and Fuzzy Logical Relationship Group (FLRG).

5. Determines Type-1 FTS defuzzification and forecasting.

6. Conducting Type-2 FTS observations. As a type-2 observation, the highest, lowest, and closing price data of the Indonesia Composite Index (ICI) will be used. The Type-2 FTS calculation has the same beginning as the Type-1 FTS calculation.



Modeling is similar to FTS Type-1, starting from determining the universe of conversations to determining the FLR and FLRG of the highest and lowest prices.

7. Define a type-2 fuzzy set.
8. Apply intersection multiple ( $\Lambda_m$ ) and union multiple ( $\vee_m$ ). Operators.

$$\begin{aligned} \vee_m &= (LHS_a, LHS_b, LHS_c, \dots) = (RHS_a \cup RHS_b \cup RHS_c \cup \dots) \\ \Lambda_m &= (LHS_a, LHS_b, LHS_c, \dots) = (RHS_a \cap RHS_b \cap RHS_c \cap \dots) \end{aligned}$$

with  $LHS_a, LHS_b, LHS_c, \dots$  and  $RHS_a \cap RHS_b \cap RHS_c \cap \dots$  are the Left-Hand Side (LHS) and Right-Hand Side (RHS) of  $FLRG_{a,b,c,\dots}$ . If there is a condition where the calculation produces an empty set value ( $\emptyset$ ), then the value will be replaced with the LHS in the Type-1 FTS variable.

$$\Lambda_m (LHS_a, LHS_b, LHS_c, \dots) = \emptyset = LHS_x$$

where  $LHS_x$  is the LHS of Type-1 FTS.

9. Perform Type-2 FTS forecasting. Type-2 FTS forecasting is obtained by finding the average value of forecasting with the defuzzified union multiple and intersection multiple operators.

$$defuzzification(t) = \frac{\sum_{q=1}^r defuzzification_q(t)}{r}$$

where  $r$  is the total of Type-2 FTS variables.

10. Evaluation of forecasting performance.

The forecasting performance error is evaluated using the Average Forecasting Error Rate (AFER) with the formula:

$$AFER = \frac{\sum_{i=1}^n \frac{|X_i - F_i|}{X_i}}{n} \times 100\%$$

where  $X$  is actual data,  $F$  is forecasting data ( $1 \leq i \leq 99$ ) and  $n$  is the total observations at time  $t$ , with AFER criteria shown in the following table [6]:

**Table 1. AFER Criteria**

AFER Value	Criteria
< 10%	Very good
10% – 20%	Good
20% – 50%	Good enough
> 50%	Bad

Therefore, the accuracy of the prediction results can be calculated using the following formula [15]:

$$Forecasting\ accuracy = 100\% - AFER$$



The flowchart of this research is shown in Fig.1 below :

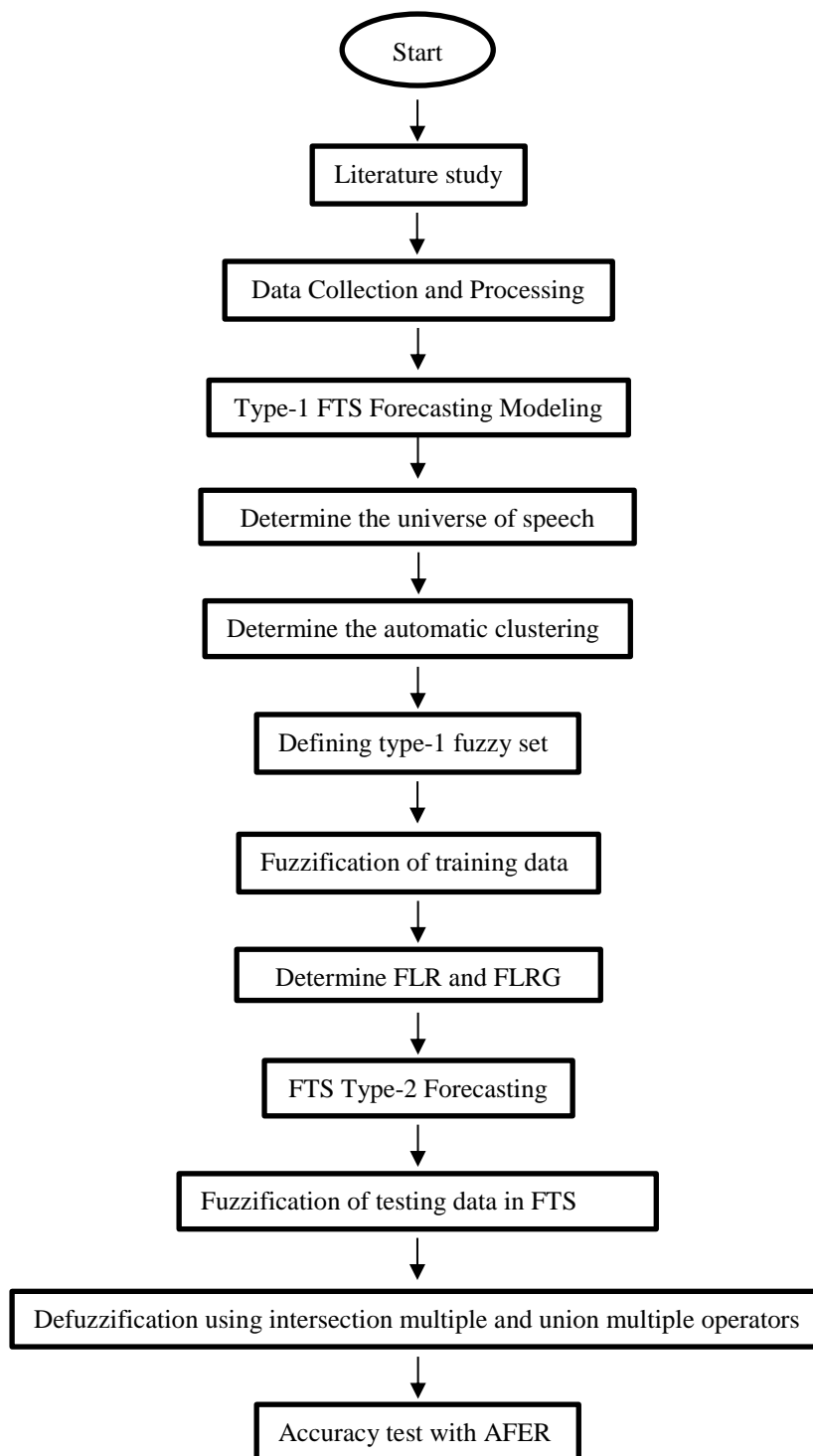
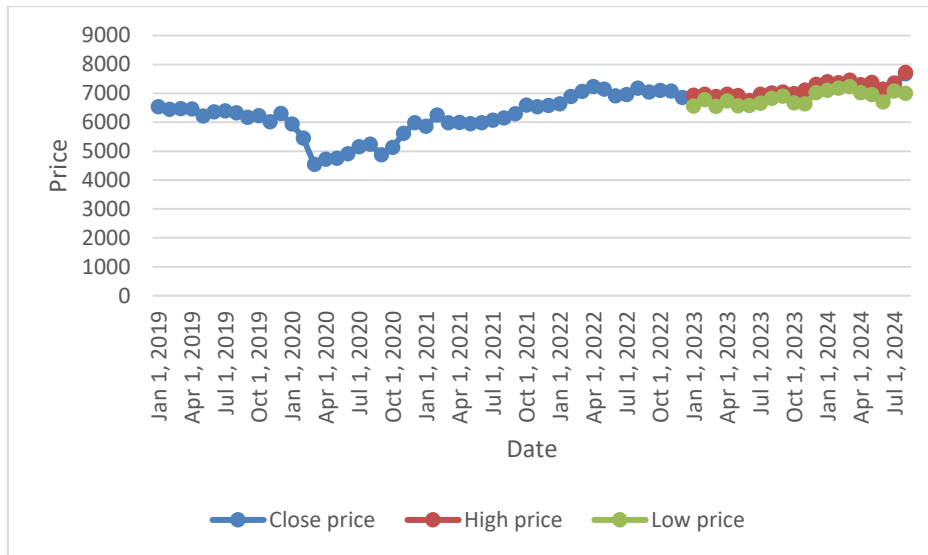


Figure 1. Research Flow Chart



**RESULTS AND DISCUSSION**

Training data is data used in Type-1 FTS in the form of closing prices consisting of 67 data from January 2019 - July 2024, while the testing data used in the Type-2 FTS process amounted to 19 data from January 2023 - July 2024. The data is depicted in Figure 2.



**Figure 2. Training and testing data on Type-1 FTS and Type-2 FTS**

Before forecasting with the Type-2 FTS model, the first step is to perform forecasting using the Type-1 FTS as follows:

1. Determine the universe of speech from historical data [14]:

$$U = [D_{min} - D_1, D_{max} + D_2]$$

The smallest and largest data based on JCI price data are 4539 and 7671. Then the universe of discussion can be obtained as follows:

$$U = [D_{min} - D_1, D_{max} + D_2] = [4539 - 9, \quad 7671 + 9]$$

$$U = [4530, \quad 7680]$$

2. Determining the length and number of intervals using the automatic clustering method and averaging over closing price variables.

The calculation of determining the length of the interval and the number of intervals with the automatic clustering algorithm was carried out using RStudio software and obtained a total of 53 intervals as in Table 2 below:

**Table 2. Intervals using the Automatic Clustering Algorithm**

Interval	Lower Boundary	Upper Boundary	Midpoint Value
$u_1$	4530	4586	4558
$u_2$	4586	4716	4651
$u_3$	4716	4754	4735
$u_4$	4754	4870	4812
⋮	⋮	⋮	
$u_{53}$	7623	7680	7651.5

3. Defining type-1 fuzzy set

After getting the intervals in Table 2, the type-1 fuzzy set can be obtained as follows:



$$\begin{aligned}
 A_1 &= \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \dots + \frac{0}{u_{53}} \\
 A_2 &= \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \dots + \frac{0}{u_{53}} \\
 A_3 &= \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \dots + \frac{0}{u_{53}} \\
 &\vdots \\
 A_{53} &= \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \dots + \frac{0}{u_{52}} + \frac{1}{u_{53}}
 \end{aligned}$$

4. Determine the Fuzzy Logical Relationship (FLR) and Fuzzy Logical Relationship Group (FLRG).  
 The closing price of the Indonesia Composite Index will be inserted into the corresponding fuzzy set. For example, in January 2019, the closing price of the stock is 6533, then the price will be included in  $A_{31}$  which is in the interval [6533, 6581]. The fuzzification results for stock closing prices can be seen in Table 3 below:

**Table 3. Type-1 FTS closing price fuzzification results**

Date	Close price	Fuzzification
January 2019	6533	$A_{31}$
February 2019	6443	$A_{29}$
March 2019	6469	$A_{30}$
April 2019	6455	$A_{29}$
May 2019	6209	$A_{25}$
⋮	⋮	⋮
July 2024	7256	$A_{51}$

Before determining the FLRG, it is necessary to determine the FLR based on Table 3. The FLR results are in Table 4 below:

**Table 4. FLR using an automatic clustering algorithm**

Date	Close price	Fuzzification	FLR
January 2019	6533	$A_{31}$	$\rightarrow A_{31}$
February 2019	6443	$A_{29}$	$A_{31} \rightarrow A_{29}$
March 2019	6469	$A_{30}$	$A_{29} \rightarrow A_{30}$
April 2019	6455	$A_{29}$	$A_{30} \rightarrow A_{29}$
May 2019	6209	$A_{25}$	$A_{29} \rightarrow A_{25}$
⋮	⋮	⋮	⋮
July 2024	7256	$A_{51}$	$A_{45} \rightarrow A_{51}$

After obtaining the FLR, the FLRG value is obtained as shown in Table 5.  
 Table 5. FLRG using automatic clustering algorithm



**Table 5. FLRG using an automatic clustering algorithm**

FLRG
$A_1 \rightarrow A_3$
$A_3 \rightarrow A_4$
$A_4 \rightarrow A_6$
$A_5 \rightarrow A_7$
$A_6 \rightarrow A_8$
$A_7 \rightarrow A_{13}$
⋮
$A_{52} \rightarrow A_{51}$

- The next step is to make observations using Fuzzy Time Series (FTS) Type-2. In this type of observation. The highest, lowest, and closing price data of the Indonesia Composite Index will be used. The fuzzification process on FTS Type-2 data means converting the lowest, highest, and closing price numbers into fuzzy values based on predetermined categories. Fuzzification is performed for data from January 2023 - July 2024. The fuzzification results are in Table 6. In the table, each price for that time period is classified into fuzzy categories that describe the price conditions on those dates.

**Table 6. Fuzzification of CSPI lowest, highest, and closing price data**

Date	Close $X_t$	Fuzzification	High $Y_t$	Fuzzification	Low $Z_t$	Fuzzification
Jan 2023	6839	$A_{37}$	6933	$A_{41}$	6558	$A_{31}$
Feb 2023	6843	$A_{37}$	6962	$A_{43}$	6781	$A_{35}$
Mar 2023	6805	$A_{37}$	6890	$A_{39}$	6543	$A_{31}$
Apr 2023	6916	$A_{40}$	6972	$A_{44}$	6735	$A_{35}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
July 2024	7256	$A_{51}$	7354	$A_{52}$	7075	$A_{45}$

- Calculate defuzzification using intersection multiple ( $\Lambda_m$ ) and union multiple ( $V_m$ ) operators.

$$V_m = (LHS_a, LHS_b, LHS_c, \dots) = (RHS_a \cup RHS_b \cup RHS_c \cup \dots)$$

$$\Lambda_m = (LHS_a, LHS_b, LHS_c, \dots) = (RHS_a \cap RHS_b \cap RHS_c \cap \dots)$$

with  $LHS_a, LHS_b, LHS_c, \dots$  and  $RHS_a \cap RHS_b \cap RHS_c \cap \dots$  are the Left-Hand Side (LHS) and Right-Hand Side (RHS) of  $FLRG_{a,b,c,\dots}$ . The defuzzification result is calculated by averaging the center values  $m_{q1}, m_{q2}, m_{q3}, \dots, m_{qj}$  in Table 2 for each interval  $u_{q1}, u_{q2}, u_{q3}, \dots, u_{qj}$ . The defuzzification results using the intersection multiple ( $\Lambda_m$ ) operator are in Table 7 and the union multiple ( $V_m$ ) defuzzification results are in Table 8.

**Table 7. Forecasting Using Intersection Multiple ( $\Lambda_m$ ) Operator**

Time (t)	Forecasting	FLRG	Forecasting ( $\Lambda_m$ )	Defuzzification
Jan 2023	X data	$A_{37} \rightarrow A_{37}, A_{40}$	$A_{37}$	6828
	Y data	$A_{41} \rightarrow A_{43}$		
	Z data	$A_{31} \rightarrow A_{29}, A_{32}$		
Feb 2023	X data	$A_{37} \rightarrow A_{37}, A_{40}$	$A_{37}$	6828
	Y data	$A_{43} \rightarrow A_{42}, A_{48}$		
	Z data	$A_{35} \rightarrow A_{45}$		
⋮	⋮	⋮	⋮	⋮





July 2024	X data	$A_{51} \rightarrow A_{49}, A_{50}, A_{53}$	$A_{51}$	7286
	Y data	$A_{52} \rightarrow A_{51}$		
	Z data	$A_{45} \rightarrow A_{38}, A_{46}, A_{49}, A_{51}$		

Table 8. Forecasting Using Union Multiple Operators ( $V_m$ )

Time (t)	Forecasting	FLRG	Forecasting ( $V_m$ )	Defuzzification
Jan 2023	X data	$A_{37} \rightarrow A_{37}, A_{40}$	$A_{29}, A_{32}, A_{37}, A_{40}, A_{43}$	6755.6
	Y data	$A_{41} \rightarrow A_{43}$		
	Z data	$A_{31} \rightarrow A_{29}, A_{32}$		
Feb 2023	X data	$A_{37} \rightarrow A_{37}, A_{40}$	$A_{37}, A_{40}, A_{42}, A_{45}, A_{48}$	6992.1
	Y data	$A_{43} \rightarrow A_{42}, A_{48}$		
	Z data	$A_{35} \rightarrow A_{45}$		
⋮	⋮	⋮	⋮	⋮
July 2024	X data	$A_{51} \rightarrow A_{49}, A_{50}, A_{53}$	$A_{38}, A_{46}, A_{49}, A_{50}, A_{51}, A_{53}$	7232.8
	Y data	$A_{52} \rightarrow A_{51}$		
	Z data	$A_{45} \rightarrow A_{38}, A_{46}, A_{49}, A_{51}$		

7. Calculating Type-2 FTS forecasting. Type-2 FTS forecasting is obtained by finding the average value of forecasting with the defuzzified union multiple and intersection multiple operators.

$$defuzzification(t) = \frac{\sum_{q=1}^r defuzzification_q(t)}{r}$$

The results of Type-2 FTS forecasting are shown in Table 9 below:

Table 9. Defuzzification Result

t	Defuzzification $\wedge_m$	Defuzzification $\vee_m$	Defuzzification
Jan 2023	6828	6755.6	6791.8
Feb 2023	6828	6992.1	6910.05
⋮	⋮	⋮	⋮
July 2024	7286	7232.8	7259.4

8. Evaluation of forecasting performance.

The forecasting performance error is evaluated using the Average Forecasting Error Rate (AFER) or known as Mean Average Percentage Error (MAPE) with the formula:

$$AFER = \frac{\sum_{t=1}^n \frac{|X_t - F_t|}{X_t}}{n} \times 100\%$$

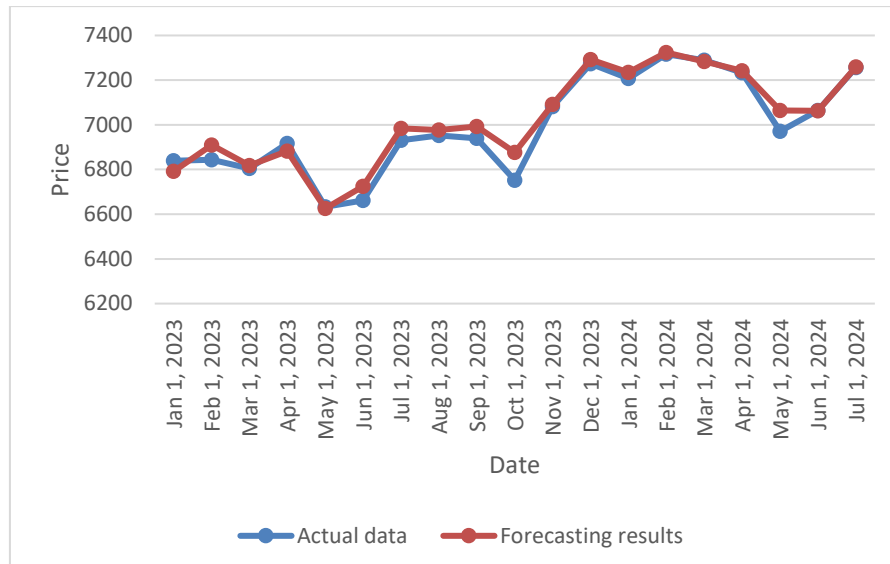
with  $X$  is actual data,  $F$  is forecasting data ( $1 \leq i \leq 99$ ) and  $n$  is the total observations at time  $t$ .

Based on the previous calculation, the AFER value is obtained as follows:

$$AFER = \frac{\sum_{t=1}^n \frac{|X_t - F_t|}{X_t}}{n} \times 100\% = \frac{0.136547}{20} \times 100\% = 0.6827$$

$$Forecasting\ accuracy = 100\% - 0.682735\% = 99.317265\% = 99.317\%$$

The AFER value obtained for ICI forecasting using type-2 FTS with an automatic clustering algorithm gives a value of 0.6827. Based on the AFER criteria table in Table 1, this result shows that the forecasting results using FTS Type-2 with an automatic clustering algorithm are very good because it has an AFER value below 10%. In addition, the accuracy of forecasting results using this method is 99.317%.



**Figure 3. Comparison plot of actual data and forecasting results on testing data**

Figure 3 displays a plot comparing the actual data with the ICI forecasting results using FTS Type 2 and an automatic clustering algorithm using Microsoft Office Excel software. Based on this, it can be observed that the actual data values and forecasting results are not much different, for example, the ICI price in July 2024 has a difference of 3.417 with actual data of 7256 and forecasting results of 7259.417.

## CONCLUSION

Forecasting results using the Indonesia Composite Index (ICI) in August 2024 amounted to 7259,417. In addition, the use of the automatic clustering algorithm to determine the length of the Type-2 Fuzzy Time Series interval is able to produce forecasts with very good accuracy. This can be seen based on the AFER value of 0.682735% which states that the performance of Type-2 FTS forecasting using the automatic clustering algorithm is very good because the AFER value is below 10% and the forecasting accuracy value is 99.317%.

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