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Product of Categories and Product of Two Objects in a Category

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ABSTRACT: Let $\mathcal{C}$ and $\mathcal{D}$ be categories. The product of two categories $\mathcal{C}$ and $\mathcal{D}$, is represented by $\mathcal{C} \times \mathcal{D}$ and referred to as the product category. Product category is an extension of the concept of the product of two sets of cartesian and is used to define the bifungtors. In the theory of categories there are also product of two objects in a category where the objects of the product itself is part of the category. The objects $A \times B$ in $\mathcal{C}$ is said to be a product of A and B in $\mathcal{C}$ if to each object $C \in \mathcal{C}$ and to each pair (f, g) of morphisms with $f : C \longrightarrow A$ and $g: C \longrightarrow B$, there is an exactly morphism $h : C \longrightarrow A \times B$, such that $f = P_A \circ h$ and $g = P_B \circ h$. This article discusses the product categories and the product of two objects in a category, including concepts and properties related to the product of categories and the product of two objects in a category.

KEYWORDS: Category, Group, Object, Morphism, Product.

INTRODUCTION

The category theory is a part of the abstract algebra that was first introduced by Eilenberg and Maclane in his writings entitled "General Theory of Natural Equivalences" in 1945 [1]. The results of the research in [1] are intended to provide an abstraction of the definitions of fungtors and natural transformations. For 15 years since its publication [1] in 1945, the concept of category theory was not considered to be a very helpful concept in the development of mathematics.

The ability of category theory to facilitate discourse widely makes mathematician Simmons say in [2], "Every mathematicians need to know at least about the existence of the theory of categories, and many of them will need to use the terms category."

Category theory is formed by the concepts of "object" and "morphism" from the underlying fields of mathematics, such as algebra or topology. Of course, there is a tendency to make the theory of categories not dependent on other branches of mathematics. However, many different fields of Mathematics can be interpreted as categories, as well as techniques and theorems of category theory can be applied to those fields.

In category theory, product of the two categories $\mathcal{C}$ and $\mathcal{D}$, is represented by $\mathcal{C} \times \mathcal{D}$ and is called the product category. Product category is an extension of the concept of the products of two sets of cartesian and is used to define the bifungtors.

In this case, it will be discussed about the product category and product two objects in a category, including concepts and properties related to the category of products and product two object in a category.

METHOD

This research is largely a literature study or a library review of sources from printed books or e-books dealing with category theory. To study and explain product categories and products of two objects in a category, it is necessary to know the theory of categories as well as the components associated with the category. To do the research, some of the procedures performed can be seen in the following picture.

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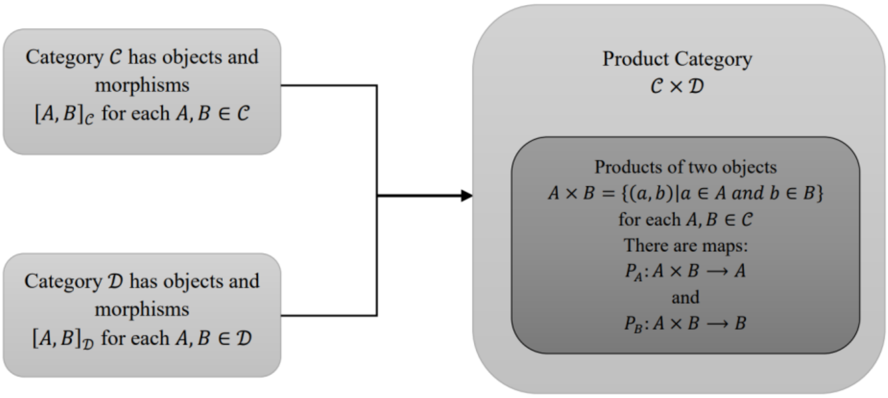

Picture 1. Diagram of research

RESULTS AND DISCUSSION

Category theory is a general theory of mathematical structures and their relationships abstractly. A category is formed by "objects" and "morphisms". Objects are the elements that form an assembly, whereas morphism can be understood as something that connects one object with another.

1.1 Product of Categories

In mathematics in particular the theory of categories, the product of two categories $\mathcal{C}$ and $\mathcal{D}$, is represented by $\mathcal{C} \times \mathcal{D}$ called product categories which are developers of more abstract related cartesian products on the assembly. Here is the definition of the product category.

Definition [3] *Let C and D be categories. The product C×D is defined as follows.*

i. $\mathcal{C} \times \mathcal{D} = \{(A,B) | A \in \mathcal{C} \text{ and } B \in \mathcal{D}\}$

ii. *For each* $(A,B) \in \mathcal{C} \times \mathcal{D}$ *there is a morphism that connects objects* (A,A') *in category* $\mathcal{C}$ *to objects* (B,B') *in category* $\mathcal{D}$
$[(A,B),(A',B')]_{\mathcal{C}\times\mathcal{D}} = [A,A']_{\mathcal{C}} \times [B,B']_{\mathcal{D}}$

iii. *Morphism composition is associative*
$$(f',g') \circ (f,g) = (f' \circ f, g' \circ g)$$

iv. *There is an indentity morphism*
$$I_{(\mathcal{C}\times\mathcal{D})} = (I_{\mathcal{C}}, I_{\mathcal{D}})$$

Then, canonical projections are obtained as follows.

$$\mathcal{C} \xleftarrow{\pi_1} \mathcal{C} \times \mathcal{D} \xrightarrow{\pi_2} \mathcal{D}$$

$\pi_1: \mathcal{C} \times \mathcal{D} \longrightarrow \mathcal{C}$
$\pi_1(A,B) = A$
$\pi_1(f,g) = f$
and
$\pi_2: \mathcal{C} \times \mathcal{D} \longrightarrow \mathcal{D}$
$\pi_2(A,B) = B$
$\pi_2(f,g) = g$

Lemma. *Let* $\mathcal{C} \times \mathcal{D}$ *be product categories,* $\mathcal{S}$ *be a subcategory in* $\mathcal{C}$*, and* $\mathcal{T}$ *be a subcategory in* $\mathcal{D}$*. Then,* $\mathcal{S} \times \mathcal{T}$ *subcategory in* $\mathcal{C} \times \mathcal{D}$.

Proof.

Let $\mathcal{S}$ be a subcategory in $\mathcal{C}$, then the objects in $\mathcal{S}$ are $|\mathcal{S}| \subseteq |\mathcal{C}|$. Whereas, for each morphism in $\mathcal{S}$ a subset of the morphism in $\mathcal{C}$, $[A,B]_{\mathcal{S}} \subseteq [A,B]_{\mathcal{C}}$ Therefore, morphism composition that applies to $\mathcal{C}$ also applies to $\mathcal{S}$.

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Let $\mathcal{T}$ be a subcategory in $\mathcal{D}$, then the objects in $\mathcal{T}$ are $|\mathcal{T}| \subseteq |\mathcal{D}|$. Whereas, for each morphism in $\mathcal{T}$ a subset of the morphism in $\mathcal{D}$, $[A,B]_{\mathcal{T}} \subseteq [A,B]_{\mathcal{D}}$ Therefore, morphism composition that applies to $\mathcal{D}$ also applies to $\mathcal{T}$.
Thus, $\mathcal{S} \times \mathcal{T}$ subcategory in $\mathcal{C} \times \mathcal{D}$.

Example 1

1) Let $\mathcal{G}rp_1$ be a category with group objects and function morphisms.
2) Let $\mathcal{G}rp_2$ be a category with group objects and function morphisms.

The product $\mathcal{G}rp_1 \times \mathcal{G}rp_2$ is defined as follows.

i. $\mathcal{G}rp_1 \times \mathcal{G}rp_2 = \{(A,B) | A \in \mathcal{G}rp_1 \; and \; B \in \mathcal{G}rp_2\}$.

ii. For each (A, B) $\in \mathcal{G}rp_1 \times \mathcal{G}rp_2$ there is a morphism that connects objects (A, A') in category $\mathcal{G}rp_1$ to objects (B, B') in category $\mathcal{G}rp_2$.
$[(A,B),(A',B')]_{\mathcal{G}rp_1 \times \mathcal{G}rp_2} = [A,A']_{\mathcal{G}rp_1} \times [B,B']_{\mathcal{G}rp_2}$
For any $f \in [(A_1, A_2)]_{\mathcal{G}rp_1}$ with the function $f: A_1 \longrightarrow A_2$, then for each $a \in A_1 \Rightarrow f(a) \in A_2$.
For any $a_1, a_2 \in [(A_1, A_2)]_{\mathcal{G}rp_1}$ with $a_1 = a_2$, then $f(a_1) = f(a_2)$.
Thus, function f is a binary operation (closed and well-defined)

iii. Morphism composition is defined as follows.
$$(f', g') \circ (f, g) = (f' \circ f, g' \circ g)$$
Let $f_1: A_1 \longrightarrow A_2$ and $f_2: A_2 \longrightarrow A_3$ be functions for all $f_1, f_2 \in \mathcal{G}rp_1$. Let $g_1: B_1 \longrightarrow B_2$ and $g_2: B_2 \longrightarrow B_3$ be relations for all $g_1, g_2 \in \mathcal{G}rp_2$. Then function composition is $f_2 \circ f_1: A_1 \longrightarrow A_3$ and relation composition is $g_2 \circ g_1: B_1 \longrightarrow B_3$.
For any $(a_1, b_1) \in A_1 \times B_1$ and $(a_2, b_2) \in A_2 \times B_2$, then
$(f_1, g_1): A_1 \times B_1 \longrightarrow A_2 \times B_2$
$$(a_1, b_1) \longmapsto \big(f_1(a_1), g_1(b_1)\big)$$
and
$(f_2, g_2): A_2 \times B_2 \longrightarrow A_3 \times B_3$
$$(a_2, b_2) \longmapsto \big(f_2(a_2), g_2(b_2)\big)$$
Then, canonical projections are defined as follows.
$$\mathcal{G}rp_1 \xleftarrow{\pi_1} \mathcal{G}rp_1 \times \mathcal{G}rp_2 \xrightarrow{\pi_2} \mathcal{G}rp_2$$
$\pi_1: \mathcal{G}rp_1 \times \mathcal{G}rp_2 \longrightarrow \mathcal{G}rp_1$
$\pi_1(f, g) = f$
and
$\pi_2: \mathcal{G}rp_1 \times \mathcal{G}rp_2 \longrightarrow \mathcal{G}rp_2$
$\pi_2(f, g) = g$
From canonical projections are obtained:
1. $\pi_1(f_1, g_1) = f_1$
$\big(f_1(a_1), g_1(b_1)\big) = f_1(a_1)$
2. $\pi_1{}'(f_2, g_2) = f_2$
$\big(f_2(a_2), g_2(b_2)\big) = f_2(a_2)$
3. $\pi_2(f_1, g_1) = g_1$
$\big(f_1(a_1), g_1(b_1)\big) = g_1(b_1)$
4. $\pi_2(f_2, g_2) = g_2$
$\big(f_2(a_2), g_2(b_2)\big) = g_2(b_2)$

Thus,
$$\Big(\big(f_2(a_2), g_2(b_2)\big) \circ \big(f_1(a_1), g_1(b_1)\big)\Big) = \big(f_2(a_2) \circ f_1(a_1), g_2(b_2) \circ g_1(b_1)\big)$$

iv. Morphism identity is defined as follows.

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$I_{(\mathcal{Grp}_1 \times \mathcal{Grp}_2)} = (I_{\mathcal{Grp}_1}, I_{\mathcal{Grp}_2})$

There is a function $I_{\mathcal{Grp}_1}: I_{\mathcal{Grp}_1} \longrightarrow I_{\mathcal{Grp}_1}$ with $I_{\mathcal{Grp}_1}(a) = a$, for each $a \in \mathcal{Grp}_1$. Such that for any functions $f_1: A_1 \longrightarrow A_2$ dan $f_2: A_2 \longrightarrow A_3$, then

$(f_2 \circ I_{\mathcal{Grp}_1})(a_2) = f_2(I_{\mathcal{Grp}_1})(a_2)$
$= f_2(a_2)$

, for each $a_2 \in A_2$

$(I_{\mathcal{Grp}_1} \circ f_1)(a_1) = I_{\mathcal{Grp}_1}(f_1)(a_1)$
$= f_1(a_1)$

, for each $a_1 \in A_1$

Thus, $I_{\mathcal{Grp}_1}$ is a morphism identity for $\mathcal{Grp}_1$.

There is a relation $I_{\mathcal{Grp}_2}: I_{\mathcal{Grp}_2} \longrightarrow I_{\mathcal{Grp}_2}$ with $I_{\mathcal{Grp}_2}(b) = b$, for each $b \in \mathcal{Grp}_2$. Such that for any relations $g_1: B_1 \longrightarrow B_2$ dan $g_2: B_2 \longrightarrow B_3$, then

$(g_2 \circ I_{\mathcal{Grp}_2})(b_2) = g_2(I_{\mathcal{Grp}_2})(b_2)$
$= g_2(b_2)$

, for each $b_2 \in B_2$

$(I_{\mathcal{Grp}_2} \circ g_1)(b_1) = I_{\mathcal{Grp}_2}(g_1)(b_1)$
$= g_1(b_1)$

, for each $b_1 \in B_1$

Thus, $I_{\mathcal{Grp}_2}$ is a morphism identity for $\mathcal{Grp}_2$.

Example 2

1) Let $\mathcal{Grp}_1$ be a category with group objects and function morphisms.
2) Let $\mathcal{Grp}_3$ be a category with group objects and group homomorphism morphisms.

The product $\mathcal{Grp}_1 \times \mathcal{Grp}_3$ is defined as follows.

i. $\mathcal{Grp}_1 \times \mathcal{Grp}_3 = \{(A, B) | A \in \mathcal{Grp}_1 \text{ and } B \in \mathcal{Grp}_3\}$

ii. For each $(A, B) \in \mathcal{Grp}_1 \times \mathcal{Grp}_3$ there is a morphism that connects objects (A, A') in category $\mathcal{Grp}_1$ to objects (B, B') in category $\mathcal{Grp}_3$.

$[(A, B), (A', B')]_{\mathcal{Grp}_1 \times \mathcal{Grp}_3} = [A, A']_{\mathcal{Grp}_1} \times [B, B']_{\mathcal{Grp}_3}$

For any $f \in [(A_1, A_2)]_{\mathcal{Grp}_1}$ with the function $f: A_1 \longrightarrow A_2$, then for each $a \in A_1 \Rightarrow f(a) \in A_2$.

For any $a_1, a_2 \in [(A_1, A_2)]_{\mathcal{Grp}_1}$ with $a_1 = a_2$, then $f(a_1) = f(a_2)$.

Thus, function f is a binary operation (closed and well-defined)

For any $g \in [(B_1, B_2)]_{\mathcal{Grp}_3}$ with group homomorphism $g: B_1 \longrightarrow B_2$, then for each $b_1, b_2 \in B_1$ obtained

$g(b_1 * b_2) = g(b_1) * g(b_2)$

Thus, g is a homomorphism group.

iii. Morphism composition is defined as follows.

$(f', g') \circ (f, g) = (f' \circ f, g' \circ g)$

Let $f_1: A_1 \longrightarrow A_2$ and $f_2: A_2 \longrightarrow A_3$ be functions for all with $f_1, f_2 \in \mathcal{Grp}_1$. Let $g_1: B_1 \longrightarrow B_2$ and $g_2: B_2 \longrightarrow B_3$ be group homomorphisms for all $g_1, g_2 \in \mathcal{Grp}_3$. Tthe function composition is $f_2 \circ f_1: A_1 \longrightarrow A_3$ and the group homomorphism composition is $g_2 \circ g_1: B_1 \longrightarrow B_3$.

Then will be proven $g_2 \circ g_1$ group homomorphism.

1. For any $b \in B_1$, then

   $g_2 \circ g_1(b) = g_2(g_1(b))$

   Thus, $g_2 \circ g_1$ is closed.

2. For any $b_1, b_2 \in B_1$ with $b_1 = b_2$, then

   $g_2 \circ g_1(b_1) = g_2 \circ g_1(b_2)$

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$$g_2(g_1(b_1)) = g_2(g_1(b_2))$$

Thus, $g_2 \circ g_1$ is well-defined.

3. For any $b_1, b_2 \in B_1$,
$g_2 \circ g_1 \in Hom(B_1, B_3)$,
$g_1 \in Hom(B_1, B_2)$, and
$g_2 \in Hom(B_2, B_3)$, then

$$\begin{aligned} g_2 \circ g_1(b_1 * b_2) &= g_2(g_1(b_1 *^1 b_2)) \\ &= g_2(g_1(b_1) *^2 g_1(b_2)) \\ &= g_2(g_1(b_1)) *^3 g_2(g_1(b_2)) \\ &= g_2 \circ g_1(b_1) *^3 g_2 \circ g_1(b_2) \end{aligned}$$

Thus, $g_2 \circ g_1$ is a group homomorphism.

And then, for each $(a_1, b_1) \in A_1 \times B_1$ and $(a_2, b_2) \in A_2 \times B_2$, obtained

$$\begin{aligned} (f_1, g_1)&: A_1 \times B_1 \longrightarrow A_2 \times B_2 \\ &(a_1, b_1) \longmapsto (f_1(a_1), g_1(b_1)) \end{aligned}$$

dan

$$\begin{aligned} (f_2, g_2)&: A_2 \times B_2 \longrightarrow A_3 \times B_3 \\ &(a_2, b_2) \longmapsto (f_2(a_2), g_2(b_2)) \end{aligned}$$

Then, canonical projections are defined as follows.

$$\mathcal{G}r\mathcal{p}_1 \xleftarrow{\pi_1} \mathcal{G}r\mathcal{p}_1 \times \mathcal{G}r\mathcal{p}_3 \xrightarrow{\pi_3} \mathcal{G}r\mathcal{p}_3$$

$\pi_1: \mathcal{G}r\mathcal{p}_1 \times \mathcal{G}r\mathcal{p}_3 \longrightarrow \mathcal{G}r\mathcal{p}_1$
$\pi_1(f, g) = f$
and
$\pi_3: \mathcal{G}r\mathcal{p}_1 \times \mathcal{G}r\mathcal{p}_3 \longrightarrow \mathcal{G}r\mathcal{p}_3$
$\pi_3(f, g) = g$

For canonical projections are obtained:

1. $\pi_1(f_1, g_1) = f_1$
$(f_1(a_1), g_1(b_1)) = f_1(a_1)$
2. $\pi_1{}'(f_2, g_2) = f_2$
$(f_2(a_2), g_2(b_2)) = f_2(a_2)$
3. $\pi_3(f_1, g_1) = g_1$
$(f_1(a_1), g_1(b_1)) = g_1(b_1)$
4. $\pi_3(f_2, g_2) = g_2$
$(f_2(a_2), g_2(b_2)) = g_2(b_2)$

Thus,

$$\big((f_2(a_2), g_2(b_2)) \circ (f_1(a_1), g_1(b_1))\big) = (f_2(a_2) \circ f_1(a_1), g_2(b_2) \circ g_1(b_1))$$

iv. Morphism identity is defined as follows.

$$I_{(\mathcal{G}r\mathcal{p}_1 \times \mathcal{G}r\mathcal{p}_3)} = (I_{\mathcal{G}r\mathcal{p}_1}, I_{\mathcal{G}r\mathcal{p}_3})$$

There is a function $I_{\mathcal{G}r\mathcal{p}_1}: I_{\mathcal{G}r\mathcal{p}_1} \longrightarrow I_{\mathcal{G}r\mathcal{p}_1}$ with $I_{\mathcal{G}r\mathcal{p}_1}(a) = a$, for each $a \in \mathcal{G}r\mathcal{p}_1$. Then, for any functions $f_1: A_1 \longrightarrow A_2$ and $f_2: A_2 \longrightarrow A_3$, obtained

$$\begin{aligned} (f_2 \circ I_{\mathcal{G}r\mathcal{p}_1})(a_2) &= f_2(I_{\mathcal{G}r\mathcal{p}_1})(a_2) \\ &= f_2(a_2) \end{aligned}$$

, for each $a_2 \in A_2$

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$\left(I_{\mathcal{Grp}_1} \circ f_1\right)(a_1) = I_{\mathcal{Grp}_1}(f_1)(a_1)$
$\qquad\qquad = f_1(a_1)$
, for each $a_1 \in A_1$
Thus, $I_{\mathcal{Grp}_1}$ is a morphism identity for $\mathcal{Grp}_1$.
For any group homomorphism $I_{\mathcal{Grp}_3} \in \mathcal{Grp}_3$ with $I_{\mathcal{Grp}_3}: I_{\mathcal{Grp}_3} \longrightarrow I_{\mathcal{Grp}_3}$. Then, for each $b_1, b_2 \in \mathcal{Grp}_3$ obtained
$I_{\mathcal{Grp}_3}(b_1 * b_2) = I_{\mathcal{Grp}_3}(b_1) * I_{\mathcal{Grp}_3}(b_2)$
$\qquad\qquad = b_1 * b_2$
Thus, $I_{\mathcal{Grp}_3}$ ia a group homomorphism.
Such that, for each group homomorphisms $g_1: B_1 \longrightarrow B_2$ and $g_2: B_2 \longrightarrow B_3$, obtained
$\left(g_2 \circ I_{\mathcal{Grp}_3}\right)(b_2) = g_2\left(I_{\mathcal{Grp}_3}\right)(b_2)$
$\qquad\qquad = g_2(b_2)$
, for each $b_2 \in B_2$
$\left(I_{\mathcal{Grp}_3} \circ g_1\right)(b_1) = I_{\mathcal{Grp}_3}(g_1)(b_1)$
$\qquad\qquad = g_1(b_1)$
, for each $b_1 \in B_1$
Thus, $I_{\mathcal{Grp}_3}$ is a morphism identity for $\mathcal{Grp}_3$.

Example 3
1) Let $\mathcal{Grp}_2$ be a category with group objects and relation morphisms.
2) Let $\mathcal{Grp}_3$ be a category with group objects and group homomorphism morphisms.

The product $\mathcal{Grp}_2 \times \mathcal{Grp}_3$ is defined as follows.

i. $\mathcal{Grp}_2 \times \mathcal{Grp}_3 = \{(A, B) | A \in \mathcal{Grp}_2 \; and \; B \in \mathcal{Grp}_3\}$

ii. For each $(A, B) \in \mathcal{Grp}_2 \times \mathcal{Grp}_3$ there is a morphism that connects objects (A, A') in category $\mathcal{Grp}_2$ to objects (B, B') in category $\mathcal{Grp}_3$.
$[(A, B), (A', B')]_{\mathcal{Grp}_2 \times \mathcal{Grp}_3} = [A, A']_{\mathcal{Grp}_2} \times [B, B']_{\mathcal{Grp}_3}$
For any $g \in [(B_1, B_2)]_{\mathcal{Grp}_3}$ with group homomorphism $g: B_1 \longrightarrow B_2$, then for each $b_1, b_2 \in B_1$ obtained
$g(b_1 * b_2) = g_1(b_2) * g(b_2)$
Thus, g is a group homomorphism.

iii. Morphism composition is defined as follows.
$$(f', g') \circ (f, g) = (f' \circ f, g' \circ g)$$
Let $f_1: A_1 \longrightarrow A_2$ and $f_2: A_2 \longrightarrow A_3$ be relations for all $f_1, f_2 \in \mathcal{Grp}_2$. Let $g_1: B_1 \longrightarrow B_2$ and $g_2: B_2 \longrightarrow B_3$ be group homomorphisms for all $g_1, g_2 \in \mathcal{Grp}_3$. The relation composition is $f_2 \circ f_1: A_1 \longrightarrow A_3$ and the group homomorphism composition is $g_2 \circ g_1: B_1 \longrightarrow B_3$.
Then will be proven $g_2 \circ g_1$ group homomorphism.
1. For any $b \in B_1$, then
$g_2 \circ g_1(b) = g_2\left(g_1(b)\right)$
Thus, $g_2 \circ g_1$ is closed.
2. For any $b_1, b_2 \in B_1$ with $b_1 = b_2$, then
$g_2 \circ g_1(b_1) = g_2 \circ g_1(b_2)$
$g_2\left(g_1(b_1)\right) = g_2\left(g_1(b_2)\right)$
Thus, $g_2 \circ g_1$ is well-defined.
3. For any $b_1, b_2 \in B_1$,
$g_2 \circ g_1 \in Hom(B_1, B_3)$,
$g_1 \in Hom(B_1, B_2)$, and
$g_2 \in Hom(B_2, B_3)$, then

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$$g_2 \circ g_1(b_1 * b_2) = g_2(g_1(b_1 *^1 b_2))$$
$$= g_2(g_1(b_1) *^2 g_1(b_2))$$
$$= g_2(g_1(b_1)) *^3 g_2(g_1(b_2))$$
$$= g_2 \circ g_1(b_1) *^3 g_2 \circ g_1(b_2)$$

Thus, $g_2 \circ g_1$ is a group homomorphism.

And then, for each $(a_1, b_1) \in A_1 \times B_1$ and $(a_2, b_2) \in A_2 \times B_2$, obtained

$$(f_1, g_1): A_1 \times B_1 \longrightarrow A_2 \times B_2$$
$$(a_1, b_1) \longmapsto (f_1(a_1), g_1(b_1))$$

dan

$$(f_2, g_2): A_2 \times B_2 \longrightarrow A_3 \times B_3$$
$$(a_2, b_2) \longmapsto (f_2(a_2), g_2(b_2))$$

Then, canonical projections are defined as follows.

$$\mathcal{Grp}_2 \xleftarrow{\pi_2} \mathcal{Grp}_2 \times \mathcal{Grp}_3 \xrightarrow{\pi_3} \mathcal{Grp}_3$$

$$\pi_2: \mathcal{Grp}_2 \times \mathcal{Grp}_3 \longrightarrow \mathcal{Grp}_2$$
$$\pi_2(f, g) = f$$

and

$$\pi_3: \mathcal{Grp}_2 \times \mathcal{Grp}_3 \longrightarrow \mathcal{Grp}_3$$
$$\pi_3(f, g) = g$$

From canonical projections are obtained:

1. $\pi_2(f_1, g_1) = f_1$
   $(f_1(a_1), g_1(b_1)) = f_1(a_1)$
2. $\pi_2{}'(f_2, g_2) = f_2$
   $(f_2(a_2), g_2(b_2)) = f_2(a_2)$
3. $\pi_3(f_1, g_1) = g_1$
   $(f_1(a_1), g_1(b_1)) = g_1(b_1)$
4. $\pi_3(f_2, g_2) = g_2$
   $(f_2(a_2), g_2(b_2)) = g_2(b_2)$

Thus,

$$\big((f_2(a_2), g_2(b_2)) \circ (f_1(a_1), g_1(b_1))\big) = (f_2(a_2) \circ f_1(a_1), g_2(b_2) \circ g_1(b_1))$$

iv. Morphism identity is defined as follows.

$$I_{(\mathcal{Grp}_2 \times \mathcal{Grp}_3)} = (I_{\mathcal{Grp}_2}, I_{\mathcal{Grp}_3})$$

There is a relation $I_{\mathcal{Grp}_2}: I_{\mathcal{Grp}_2} \longrightarrow I_{\mathcal{Grp}_2}$ with $I_{\mathcal{Grp}_2}(a) = a$, for each $a \in \mathcal{Grp}_2$. Then, for each relation $f_1: A_1 \longrightarrow A_2$ and $f_2: A_2 \longrightarrow A_3$, obtained

$$(f_2 \circ I_{\mathcal{Grp}_2})(a_2) = f_2(I_{\mathcal{Grp}_2})(a_2)$$
$$= f_2(a_2)$$

, for each $a_2 \in A_2$

$$(I_{\mathcal{Grp}_2} \circ f_1)(a_1) = I_{\mathcal{Grp}_2}(f_1)(a_1)$$
$$= f_1(a_1)$$

, for each $a_1 \in A_1$

Thus, $I_{\mathcal{Grp}_2}$ is a morphism identity for $\mathcal{Grp}_2$.

For any group homomorphism $I_{\mathcal{Grp}_3} \in \mathcal{Grp}_3$ with $I_{\mathcal{Grp}_3}: I_{\mathcal{Grp}_3} \longrightarrow I_{\mathcal{Grp}_3}$. Then, for each $b_1, b_2 \in \mathcal{Grp}_3$ obtained

$$I_{\mathcal{Grp}_3}(b_1 * b_2) = I_{\mathcal{Grp}_3}(b_1) * I_{\mathcal{Grp}_3}(b_2)$$

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$= b_1 * b_2$

Thus, $I_{\mathcal{Grp}_3}$ is a group homomorphism.

And then, for each group homomorphisms $g_1: B_1 \longrightarrow B_2$ dan $g_2: B_2 \longrightarrow B_3$, obtained

$$\left(g_2 \circ I_{\mathcal{Grp}_3}\right)(b_2) = g_2\left(I_{\mathcal{Grp}_3}\right)(b_2)$$
$$= g_2(b_2)$$

, for each $b_2 \in B_2$

$$\left(I_{\mathcal{Grp}_3} \circ g_1\right)(b_1) = I_{\mathcal{Grp}_3}(g_1)(b_1)$$
$$= g_1(b_1)$$

, for each $b_1 \in B_1$

Thus, $I_{\mathcal{Grp}_3}$ is a morphism identity for $\mathcal{Grp}_3$.

1.2 Product of Two Objects in a Category

Another important idea in category theory is the products of two objects in a category where the objects of the product themselves are part of that category, which is described as follows.

Definition [4] *Let $\mathcal{C}$ be a category and let $A, B \in \mathcal{C}$ be given. The products of the two sets A and B are written as follows.*

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$$

Then there are maps:

$$P_A: A \times B \longrightarrow A$$
$$(a, b) \mapsto a$$

and

$$P_B: A \times B \longrightarrow B$$
$$(a, b) \mapsto b$$

A triple $(A \times B, P_A, P_B)$ with $A \times B$ an object in $\mathcal{C}$ and P_A, P_B morphisms in $\mathcal{C}$ is said to be a product of A and B in $\mathcal{C}$ if to each object $C \in \mathcal{C}$ and to each pair (f, g) of morphisms with $f : C \longrightarrow A$ and $g: C \longrightarrow B$, there is a exactly morphism $h : C \longrightarrow A \times B$. Such that $f = P_A \circ h$ and $g = P_B \circ h$.

The product of two objects in a category also meets the following characteristics.

i. For each $\mathrm{C} \in \mathcal{C}$ with $\mathrm{f} : \mathrm{C} \longrightarrow \mathrm{A}$ and $\mathrm{g}: \mathrm{C} \longrightarrow \mathrm{B}$, then there is an exactly morphism $\mathrm{h} : \mathrm{C} \longrightarrow \mathrm{A} \times \mathrm{B}$. Such that

$$[\mathrm{C}, \mathrm{A} \times \mathrm{B}]_{\mathcal{C}} \cong [\mathrm{C}, \mathrm{A}]_{\mathcal{C}} \times [\mathrm{C}, \mathrm{B}]_{\mathcal{C}}$$

For each $\mathrm{h} \in [\mathrm{C}, \mathrm{A} \times \mathrm{B}]_{\mathcal{C}}$, then

$$\alpha: [\mathrm{C}, \mathrm{A} \times \mathrm{B}]_{\mathcal{C}} \longrightarrow [\mathrm{C}, \mathrm{A}]_{\mathcal{C}} \times [\mathrm{C}, \mathrm{B}]_{\mathcal{C}}$$
$$\mathrm{h} \mapsto (\mathrm{P_A} \circ \mathrm{h},\ \mathrm{P_B} \circ \mathrm{h}) = \alpha(\mathrm{h})$$

$$C \xrightarrow{h} A \times B \xrightarrow{P_A} A$$

$\mathrm{P_A} \circ \mathrm{h} \in [\mathrm{C}, \mathrm{A}]_{\mathcal{C}}$

$$C \xrightarrow{h} A \times B \xrightarrow{P_B} B$$

$\mathrm{P_B} \circ \mathrm{h} \in [\mathrm{C}, \mathrm{B}]_{\mathcal{C}}$

Such that

$(\mathrm{P_A} \circ \mathrm{h},\ \mathrm{P_B} \circ \mathrm{h}) \in [\mathrm{C}, \mathrm{A}]_{\mathcal{C}} \times [\mathrm{C}, \mathrm{B}]_{\mathcal{C}}$

And then it will be shown that α isomorphism.

1) Let α be given as follows.

$$\alpha: [\mathrm{C}, \mathrm{A} \times \mathrm{B}]_{\mathcal{C}} \longrightarrow [\mathrm{C}, \mathrm{A}]_{\mathcal{C}} \times [\mathrm{C}, \mathrm{B}]_{\mathcal{C}}$$
$$\mathrm{h} \mapsto (\mathrm{P_A} \circ \mathrm{h},\ \mathrm{P_B} \circ \mathrm{h}) = \alpha(\mathrm{h})$$

For each $\mathrm{h}_1, \mathrm{h}_2 \in [\mathrm{C}, \mathrm{A} \times \mathrm{B}]_{\mathcal{C}}$ then

$\mathrm{h}_1 * \mathrm{h}_2 \in [\mathrm{C}, \mathrm{A} \times \mathrm{B}]_{\mathcal{C}}$

Thus, α is closed.

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2) Let α be given as follows.
$\alpha: [C, A \times B]_{\mathcal{C}} \longrightarrow [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$
$h \mapsto (P_A \circ h,\ P_B \circ h) = \alpha(h)$
For each $h_1, h_2 \in [C, A \times B]_{\mathcal{C}}$ with $h_1 = h_2$, then $\alpha(h_1) = \alpha(h_2)$. Because $\alpha(h_1) = \alpha(h_2)$, then $\big(P_A \circ (h_1), P_B \circ (h_1)\big) = \big(P_A \circ (h_2), P_B \circ (h_2)\big)$. It will be shown $P_A \circ (h_1) = P_A \circ (h_2)$ and $P_B \circ (h_1) = P_B \circ (h_2)$.
For any $(f, g) \in [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$ with $f: C \longrightarrow A$ and $g: C \longrightarrow B$. For each $C \in \mathcal{C}$ then
$h: C \longrightarrow A \times B$
$c \mapsto \big(f(c), g(c)\big) = h(c)$
Will be shown that $h_1(c) = h_2(c)$, then
$$P_A \circ h_1(c) = P_A \circ h_2(c)$$
$$h_1(c) = h_2(c)$$
and
$$P_B \circ h_1(c) = P_B \circ h_2(c)$$
$$h_1(c) = h_2(c)$$
Thus, α is well-defined.

3) Let α be given as follows.
$\alpha: [C, A \times B]_{\mathcal{C}} \longrightarrow [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$
$h \mapsto (P_A \circ h,\ P_B \circ h) = \alpha(h)$
For each $h_1, h_2 \in [C, A \times B]_{\mathcal{C}}$, then
$\alpha(h_1 * h_2) = \big(P_A \circ (h_1 * h_2),\ P_B \circ (h_1 * h_2)\big)$.
For each $(f, g) \in [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$ with $f: C \longrightarrow A$ and $g: C \longrightarrow B$. For each $C \in \mathcal{C}$ then
$h: C \longrightarrow A \times B$
$c \mapsto \big(f(c), g(c)\big) = h(c)$
Such that
$$\alpha(h_1 * h_2)(c) = \big(P_A \circ (h_1 * h_2)(c),\ P_B \circ (h_1 * h_2)(c)\big)$$
$$= \big(P_A(h_1 * h_2)(c),\ P_B(h_1 * h_2)(c)\big)$$
Will be shown that $h_1(c) = h_2(c)$, then
$$P_A \circ h_1(c) = P_A \circ h_2(c)$$
$$h_1(c) = h_2(c)$$
and
$$P_B \circ h_1(c) = P_B \circ h_2(c)$$
$$h_1(c) = h_2(c)$$
Then
$$\alpha(h_1 * h_2)(c) = \big(P_A(h_1 * h_2)(c),\ P_B(h_1 * h_2)(c)\big)$$
$$= \big(P_A(h_1)(c) * P_A(h_2)(c), P_B(h_1)(c) * P_B(h_2)(c)\big)$$
$$= \big(P_A(h_1)(c) * P_A(h_1)(c), P_B(h_2)(c) * P_B(h_2)(c)\big)$$
$$= \big(2P_A(h_1)(c), 2\,P_B(h_2)(c)\big)$$
$$= 2\big(P_A(h_1)(c), P_B(h_2)(c)\big)$$
$$= \big(P_A(h_1)(c), P_B(h_2)(c)\big) * \big(\,P_A(h_1)(c), P_B(h_2)(c)\big)$$
$$= \big(P_A(h_1)(c), P_B(h_1)(c)\big) * \big(\,P_A(h_2)(c), P_B(h_2)(c)\big)$$
$$= \big(P_A \circ (h_1)(c), P_B \circ (h_1)(c)\big) * \big(\,P_A \circ (h_2)(c), P_B \circ (h_2)(c)\big)$$
$$= \alpha(h_1)(c) * \alpha(h_2)(c)$$
Thus, α is homomorphism.

4) Let α be given as follows.

$\alpha: [C, A \times B]_{\mathcal{C}} \longrightarrow [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$

$h \mapsto (P_A \circ h,\ P_B \circ h) = \alpha(h)$

for each $h_1, h_2 \in [C, A \times B]_{\mathcal{C}}$ with $\alpha(h_1) = \alpha(h_2)$ then $h_1 = h_2$. Because $\alpha(h_1) = \alpha(h_2)$, then $\left(P_A \circ (h_1), P_B \circ (h_1)\right) = \left(P_A \circ (h_2), P_B \circ (h_2)\right)$. It will be shown that $P_A \circ (h_1) = P_A \circ (h_2)$ and $P_B \circ (h_1) = P_B \circ (h_2)$.

For any $(f, g) \in [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$ with $f: C \longrightarrow A$ and $g: C \longrightarrow B$. For each $C \in \mathcal{C}$ then

$h: C \longrightarrow A \times B$

$c \mapsto \left(f(c), g(c)\right) = h(c)$

Such that

$P_A \circ h_1(c) = P_A \circ h_2(c)$

$h_1(c) = h_2(c)$

and

$P_B \circ h_1(c) = P_B \circ h_2(c)$

$h_1(c) = h_2(c)$

Thus, α is injective.

5) Let α be given as follows.

$\alpha: [C, A \times B]_{\mathcal{C}} \longrightarrow [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$

$h \mapsto (P_A \circ h,\ P_B \circ h) = \alpha(h)$

For any $(f, g) \in [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$ with $f: C \longrightarrow A$ and $g: C \longrightarrow B$, then $\alpha(h) = (f, g)$.

For each $C \in \mathcal{C}$, then

$h: C \longrightarrow A \times B$

$c \mapsto \left(f(c), g(c)\right) = h(c)$

Such that

$\alpha\left(h(c)\right) = \left(f(c), g(c)\right)$

Thus, α is surjective.

From i, ii, iii, iv, and v, then proven that α isomorphism.

ii. Morphisms P_A and P_B are called canonical projections called canonical projections or morphism projections, while h is called the morphism product of the morphisms f and g which is denoted by $\langle f, g \rangle$.

Example 4

Let $\mathcal{Grp}$ be a category and $G, H \in \mathcal{Grp}$ are given.

The direct product is defined as follows.

$G \times H = \{(a, b) | a \in G \text{ and } b \in H\}$

Then there are maps

$P_G : G \times H \longrightarrow G$

$(a, b) \mapsto a$

and

$P_H : G \times H \longrightarrow H$

$(a, b) \mapsto b$

i. For each $C \in \mathcal{Grp}$ with $f : C \longrightarrow G$ and $g : C \longrightarrow H$, then there is an exactly morphism $h : C \longrightarrow G \times H$. Such that $[C, G \times H]_{\mathcal{Grp}} \cong [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$

For each $h \in [C, G \times H]_{\mathcal{Grp}}$, then

$\alpha: [C, G \times H]_{\mathcal{Grp}} \longrightarrow [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$

$h \mapsto (P_G \circ h,\ P_H \circ h) = \alpha(h)$

$$C \xrightarrow{\ h\ } G \times H \xrightarrow{\ P_G\ } G$$

$P_G \circ h \in [C, G]_{\mathcal{Grp}}$

$$C \xrightarrow{h} G \times H \xrightarrow{P_H} H$$

$P_H \circ h \in [C, H]_{\mathcal{Grp}}$

Such that

$(P_G \circ h,\ P_H \circ h) \in [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$

Then, it will be shown that α isomorphism.

1) Let α be given as follows.

$\alpha: [C, G \times H]_{\mathcal{Grp}} \longrightarrow [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$

$h \mapsto (P_G \circ h,\ P_H \circ h) = \alpha(h)$

For each $h_1, h_2 \in [C, G \times H]_{\mathcal{Grp}}$ obtained $h_1 * h_2 \in [C, G \times H]_{\mathcal{Grp}}$

Thus, α is closed.

2) Let α be given as follows.

$\alpha: [C, G \times H]_{\mathcal{Grp}} \longrightarrow [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$

$h \mapsto (P_G \circ h,\ P_H \circ h) = \alpha(h)$

For each $h_1, h_2 \in [C, G \times H]_{\mathcal{Grp}}$ with $h_1 = h_2$, then $\alpha(h_1) = \alpha(h_2)$. Because $\alpha(h_1) = \alpha(h_2)$, then $\big(P_G \circ (h_1), P_H \circ (h_1)\big) = \big(P_G \circ (h_2), P_H \circ (h_2)\big)$. It will be shown that $P_G \circ (h_1) = P_H \circ (h_2)$ and $P_G \circ (h_1) = P_H \circ (h_2)$.

For any $(f, g) \in [C, G]_{\mathcal{C}} \times [C, H]_{\mathcal{C}}$ with $f: C \longrightarrow G$ and $g: C \longrightarrow H$. For each $C \in \mathcal{Grp}$ then

$h: C \longrightarrow G \times H$

$c \mapsto \big(f(c), g(c)\big) = h(c)$

It will be shown that $h_1(c) = h_2(c)$, obtained

$P_G \circ h_1(c) = P_G \circ h_2(c)$

$h_1(c) = h_2(c)$

and

$P_H \circ h_1(c) = P_H \circ h_2(c)$

$h_1(c) = h_2(c)$

Thus, α is well-defined.

3) Let α be given as follows.

$\alpha: [C, G \times H]_{\mathcal{Grp}} \longrightarrow [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$

$h \mapsto (P_G \circ h,\ P_H \circ h) = \alpha(h)$

For each $h_1, h_2 \in [C, G \times H]_{\mathcal{Grp}}$, then

$\alpha(h_1 * h_2) = \big(P_G \circ (h_1 * h_2),\ P_H \circ (h_1 * h_2)\big)$

For any $(f, g) \in [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$ with $f: C \longrightarrow G$ and $g: C \longrightarrow H$. For each $C \in \mathcal{Grp}$, then

$h: C \longrightarrow G \times H$

$c \mapsto \big(f(c), g(c)\big) = h(c)$

Such that

$\alpha(h_1 * h_2)(c) = \big(P_G \circ (h_1 * h_2)(c),\ P_H \circ (h_1 * h_2)(c)\big)$

$= \big(P_G(h_1 * h_2)(c),\ P_H(h_1 * h_2)(c)\big)$

It will be shown that $h_1(c) = h_2(c)$, obtained

$P_G \circ h_1(c) = P_G \circ h_2(c)$

$h_1(c) = h_2(c)$

and

$P_H \circ h_1(c) = P_H \circ h_2(c)$

$h_1(c) = h_2(c)$

Such that

$\alpha(h_1 * h_2)(c) = \big(P_G(h_1 * h_2)(c),\ P_H(h_1 * h_2)(c)\big)$
$= \big(P_G(h_1)(c) * P_G(h_2)(c), P_H(h_1)(c) * P_H(h_2)(c)\big)$
$= \big(P_G(h_1)(c) * P_G(h_1)(c), P_H(h_2)(c) * P_H(h_2)(c)\big)$
$= \big(2P_G(h_1)(c), 2\, P_H(h_2)(c)\big)$
$= 2\big(P_G(h_1)(c), P_H(h_2)(c)\big)$
$= \big(P_G(h_1)(c), P_H(h_2)(c)\big) * \big(\, P_G(h_1)(c), P_H(h_2)(c)\big)$
$= \big(P_G(h_1)(c), P_H(h_1)(c)\big) * \big(\, P_G(h_2)(c), P_H(h_2)(c)\big)$
$= \big(P_G \circ (h_1)(c), P_H \circ (h_1)(c)\big) * \big(\, P_G \circ (h_2)(c), P_H \circ (h_2)(c)\big)$
$= \alpha(h_1)(c) * \alpha(h_2)(c)$

Thus, α is homomorphism.

4) Let α be given as follows.

$\alpha: [C, G \times H]_{\mathcal{Grp}} \longrightarrow [C, G]_{\mathcal{C}} \times [C, H]_{\mathcal{Grp}}$
$h \mapsto (P_G \circ h,\ P_H \circ h) = \alpha(h)$

For each $h_1, h_2 \in [C, G \times H]_{\mathcal{C}}$ with $\alpha(h_1) = \alpha(h_2)$ then $h_1 = h_2$. Because $\alpha(h_1) = \alpha(h_2)$, then $\big(P_G \circ (h_1), P_H \circ (h_1)\big) = \big(P_G \circ (h_2), P_H \circ (h_2)\big)$. It will be shown that $P_G \circ (h_1) = P_H \circ (h_2)$ and $P_H \circ (h_1) = P_H \circ (h_2)$

For any $(f, g) \in [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$ with $f: C \longrightarrow G$ and $g: C \longrightarrow H$. For each $C \in \mathcal{Grp}$ then

$h: C \longrightarrow G \times H$
$c \mapsto \big(f(c), g(c)\big) = h(c)$

Such that

$P_G \circ h_1(c) = P_G \circ h_2(c)$
$h_1(c) = h_2(c)$

and

$P_H \circ h_1(c) = P_H \circ h_2(c)$
$h_1(c) = h_2(c)$

Thus, α is injective

5) Let α be given as follows.

$\alpha: [C, G \times H]_{\mathcal{Grp}} \longrightarrow [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$
$h \mapsto (P_G \circ h,\ P_H \circ h) = \alpha(h)$

For any $(f, g) \in [C, G]_{\mathcal{Grp}} \times [C, H]_{\mathcal{Grp}}$ with $f: C \longrightarrow G$ and $g: C \longrightarrow H$, then $\alpha(h) = (f, g)$.

For each $C \in \mathcal{Grp}$, then

$h: C \longrightarrow G \times H$
$c \mapsto \big(f(c), g(c)\big) = h(c)$

Such that

$\alpha\big(h(c)\big) = \big(f(c), g(c)\big)$

Thus, α is surjective.

From i, ii, iii, iv, and v, then proven that α isomorphism.

ii. Morphisms P_G and P_H are called canonical projections called canonical projections or morphism projections, while h is called the morphism product of the morphisms f and g which is denoted by $\langle f, g \rangle$.

CONCLUSION

Based on the description of product categories and products of two objects in a category, it is obtained that the products of two categories $\mathcal{C}$ and $\mathcal{D}$, represented by $\mathcal{C} \times \mathcal{D}$ is called the product category which is a more abstract development related to product cartesian. The products two category $\mathcal{C}$ and $\mathcal{D}$ is called a product category if it has elements of object, morphism, composition of morphisms is associative, and identity morphism.

The product of two objects in a category is the product of two objects in a category where the product object itself is part of that category. The product of two objects in a category is called a product if isomorphism applies, that is, for each $C \in \mathcal{C}$ with $f : C \longrightarrow A$ and $g: C \longrightarrow B$, then there is an exactly morphism $h : C \longrightarrow A \times B$. Such that

$$[C, A \times B]_{\mathcal{C}} \cong [C, A]_{\mathcal{C}} \times [C, B]_{\mathcal{C}}$$

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