International Journal of Current Science Research and Review
ISSN: 2581-8341
Volume 07 Issue 03 March 2024
DOI: 10.47191/ijcsrr/V7-i3-51, Impact Factor: 7.943
IJCSRR @ 2024
www.ijcsrr.org

Using Mathematical Thinking in Solving Trigonometric Problems within Mason's Cognitive Framework

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ABSTRACT: The paper focuses on the synergy between mathematical thinking and solving trigonometric problems within Mason's cognitive framework. It presents and analyzes concepts related to mathematical thinking, especially Mason's definition of the thinking process into three stages: input, impact, and evaluation. Applying this framework to solving trigonometric equations, the paper illustrates how mathematical thinking can help approach problems in an organized and rigorous manner, while opening opportunities for creativity and exploration.

Discussing mathematical thinking, the paper defines it as a creative process involving prediction, induction, interpretation, description, abstraction, and reasoning. Aligned with Mason's definition of mathematical thinking, the paper describes how this thinking aids students in understanding complex structures and solving problems quickly and flexibly.

KEYWORDS: Development of mathematical thinking, Mathematical thinking, Mason's thinking, Mason's thinking process, Thinking, Trigonometry.

INTRODUCTION

Mathematics inherently is a social activity, where a community of trained practitioners (mathematicians) engages in the science of models - systematic efforts, based on observation, research, and experimentation, to discern the nature or principles of rules within systems delineated by premises or "pure mathematical" theory or models of abstracted systems from the real-world objects of "applied mathematics." The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. Mathematical thinking is a subject of great interest to scientists worldwide. In this era, any nation is concerned about the development of mathematical thinking for students, as it forms the foundation for the development of all other sciences.

In the context of this paper, we emphasize the importance of mathematical thinking and how it relates to solving geometric problems within the framework of Mason's thinking. Mathematical thinking is not only the process of creating mathematical knowledge but also an important tool for students to deeply understand the complex structures of problems. The goal of the paper is to apply this thinking framework to solve geometric problems, an area where mathematical thinking plays a significant role in the educational process.

1. Mathematical Thinking

According to Edgar Morin (2009), "Thinking is the highest method of organizing the activities of the mind, which, through language, establishes conceptions of reality and its worldview." According to Nguyen Van Doc and colleagues (2023), "Thinking is an intellectual activity that affects the brain, reflecting cognitive intentions about phenomena objectively, to solve problems, relationships in the daily life of human society."

Henderson, Hichtner (2002), and other researchers have defined mathematical thinking as a creative process, aiming to create mathematical knowledge by connecting existing knowledge with new knowledge to solve mathematical problems. According to Liu (2003), mathematical thinking is a combination of prediction, generalization, interpretation, description, abstraction, sampling, formal and informal reasoning, verification, and similar complex processes based on definitions and explanations that have been made about mathematical thinking.

According to Mason, Burton & Stacey (2010), Mathematical thinking is a dynamic process that helps us understand complex structures more easily by combining our ideas. Schoenfeld (2014) defined "mathematical thinking" as the process of searching, problem-solving, reasoning, analyzing, and interpreting information logically and accurately.

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According to Nguyen Van Doc and colleagues (2023), mathematical thinking is a complex cognitive process, including many different thinking activities such as searching, reasoning, analyzing, and interpreting information logically and accurately, helping students use mathematical knowledge and skills to solve problems effectively, including daily life problems.

Thus, mathematical thinking not only helps students solve problems but also serves as a comprehensive approach to understanding and solving problems in daily life and research. It plays an important role in developing thinking and working skills in complex environments.

2. Geometry Mathematics Curriculum

According to MOET (2018), the Mathematics curriculum helps students achieve the following main objectives:

- Develop and enhance mathematical abilities including the following core elements: mathematical thinking and reasoning abilities; mathematical modeling abilities; mathematical problem-solving abilities; mathematical communication abilities; and the ability to use mathematical tools and resources.
- Contribute to the development of students' key qualities and general competencies at appropriate levels as specified in the comprehensive curriculum.
- Acquire common, fundamental, and essential mathematical knowledge and skills; develop the ability to solve interdisciplinary problems between Mathematics and other subjects such as Physics, Chemistry, Biology, Geography, Informatics, Technology, History, Art, etc.; provide opportunities for students to experience and apply mathematics in practice.
- Have a relatively general understanding of the usefulness of mathematics for each related profession to guide career orientation, as well as have sufficient minimum capability to self-study mathematics-related issues throughout life.

The Geometry Mathematics curriculum: According to the curriculum of the Ministry of Education and Training of Vietnam, the geometry topic is distributed in grades 9, 10, and 11.

- Grade 9: Trigonometric ratios in right triangles with content on trigonometric ratios of acute angles. Some formulas about sides and angles in right triangles.
- Grade 10: Trigonometric ratios in triangles, vectors with content on trigonometric ratios in triangles, cosine law, sine law, formulas for calculating triangle area and solving triangles. Vectors, operations (sum and difference of two vectors, scalar product of a number with a vector, dot product of two vectors), and some applications in Physics.
- Grade 11: Trigonometric functions and equations with content on trigonometric angles, angle measures, trigonometric circles, trigonometric values (trigonometric angles, relationships between trigonometric values), trigonometric transformations (sum formula; double angle formula; product-to-sum formula; sum-to-product formula) and Trigonometric functions and graphs.

3. Mason's Framework of Thinking

The thinking process when solving mathematical problems can be traced through three stages of mathematical thinking according to Mason (2010). These three stages include the initiation stage, the engagement stage, and the verification stage.

In the initiation stage: the problem is identified by carefully reading the question, identifying related ideas, focusing on their own discoveries, classifying concise information, and representing it in symbols or symbols.

The engagement stage: is marked by the appearance of assumptions or hypotheses showing confidence in a statement.

The verification stage: is marked by activities to check the accuracy of the solution, the solution's relevance to the question, providing reasons to ensure an appropriate solution, and the consequences of the hypothesis or argument.

Table 1. Mason Framework

Stages	Self-Evaluation	Activities
	I know	Carefully read the question Specialize to explore what is relevant Which ideas/skills/events are appropriate? Do I know any similar questions?

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Initiation	I want	Organize and condense information Guard against vagueness Specialize to explore what the question really is?
	Introduction	Pictures, diagrams, symbols Representations, symbols, organization
Engagement	Prediction	Cyclic process
	May	Systematically specialize Types of inference
	Why	Justification Calculation
Verification	Checking	Argue to ensure that the calculations are appropriate Examine the results of conclusions to see if they are reasonable Ensure the solution is appropriate to the question
	Why	Justification
	Reflection	On main ideas and points On the consequences of hypotheses and arguments On your solution: can you make it clearer?
	Expansion	Expand the results into a broader context by generalizing By seeking a new approach to solving By changing some limitations

4. Applying Mason's framework of thinking to solve trigonometric equations

a. Trigonometric Equations

According to the curriculum of the Ministry of Education and Training of Vietnam (2018), trigonometric equations are taught and learned in grade 11. With the following achievements:

- Recognizing the solution formulas of basic trigonometric equations: sin x = m; cos x = m; tan x = m; cot x = m by applying the graphs of corresponding trigonometric functions.
- Approximating the solutions of basic trigonometric equations using handheld calculators.
- Solving trigonometric equations in the form of directly applying basic trigonometric equations (e.g., solving trigonometric equations like sin 2x = sin 3x, sin x = cos 3x).
- Solving some practical problems related to trigonometric equations (e.g., some problems related to harmonic oscillation in Physics,...)

Thus, students need to apply the knowledge they have learned to solve complex problems by simplifying them into basic problems they have learned to solve their problems through mathematical thinking.

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IJCSRR @ 2024

www.ijcsrr.org

b. Using Mason's framework of thinking to solve trigonometric equations in the form: asinx + bcosx = c

Stages	**Self-Evaluation**	**Activities**
Initiation	I know	Read the question carefully, this is a first order trigonometric equation for sinx and cosx. asinx + bcosx = c + Specializes in discovering related things: related trigonometric formulas $\sin(a+b) = \sin a.\cos b + \sin b.\cos a$ $\sin(a-b) = \sin a.\cos b - \sin b.\cos a$ $\cos(a+b) = \cos a.\cos b - \sin a.\sin b$ $\cos(a-b) = \cos a.\cos b + \sin a.\sin b$ $\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a.\tan b}$ $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a.\tan b}$ +What ideas/skills/facts are appropriate? sin(a+b) = sina.cosb + sinb.cosa or sin(a-b) = sina.cosb - sinb.cosa +Do I know any similar questions? problems about solving first-order equations For example: $\sqrt{3}\cos x - \sin x = \sqrt{2}$ $\cos x - \sqrt{3}\sin x = -1$
	I want	+ Classify and summarize information For the equation : asinx + bcosx = c Set cosx= $\dfrac{a}{\sqrt{a^2+b^2}}$; sinx= $\dfrac{b}{\sqrt{a^2+b^2}}$ $\Rightarrow \sqrt{a^2+b^2}\sin(x+\alpha) = c$ Beware of ambiguity + Specialized to discover what the real question is? Solve the equation : $\sqrt{3}\cos x - \sin x = \sqrt{2}$
	Introduction	The mathematical symbols used when solving first-order trigonometric equations for trigonometric functions are: = ; ⇒; ⇔; ≥; <; …

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Engagement	Prediction	Cyclic process
	May	For the equation :asinx + bcosx = c $\Rightarrow \sqrt{a^2+b^2}\sin(x+\alpha)=c$
	Why	Return to basic trigonometric equation form sinx = a.
Verification	Checking	The condition for the equation to have a solution is: $a^2+b^2 \geq c^2$
	Why	Justification: $a^2 + b^2 <0$ then the equation has no solution
	Reflection	This is a general solution for all first-order equations asinx + bcosx = c
	Expansion	**Method 1 :** asinx + bcosx = c Set cosx= $\frac{a}{\sqrt{a^2+b^2}}$; sinx= $\frac{b}{\sqrt{a^2+b^2}}$ $\Rightarrow \sqrt{a^2+b^2}\sin(x+\alpha)=c$ **Method 2 :** $a\left[\sin x+\frac{b}{a}\cos x\right]=c$ Set : $\frac{b}{a}=\tan\alpha \Rightarrow a\left[\sin x+\cos x.\tan\alpha\right]=c$ $\Leftrightarrow \sin(x+\alpha)=\frac{c}{a}\cos\alpha$ **Method 3 :** Set : $t=\tan\frac{x}{2}$ we have : $\sin x=\frac{2t}{1+t^2};\ \cos x=\frac{1-t^2}{1+t^2}$ $\Rightarrow (b+c)t^2-2at-b+c=0$

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c. Example of using Mason's framework of thinking to solve trigonometric equations: 3sinx + 4cosx = 5

Stages	**Self-Evaluation**	**Activities**
Initiation	I know	+ Read the question carefully, this is a first-order trigonometric equation for sinx and cosx. asinx + bcosx = c + Specializes in discovering related things: related trigonometric formulas $\sin(a+b) = \sin a.\cos b + \sin b.\cos a$ $\sin(a-b) = \sin a.\cos b - \sin b.\cos a$ $\cos(a+b) = \cos a.\cos b - \sin a.\sin b$ $\cos(a-b) = \cos a.\cos b + \sin a.\sin b$ $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a.\tan b}$ $\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a.\tan b}$ +What ideas/skills/facts are appropriate? sin(a+b) = sina.cosb + sinb.cosa or sin(a-b) = sina.cosb - sinb.cosa +Do I know any similar questions? Answer: yes.
	I want	+ Classify and summarize information For the equation 3sinx + 4cosx = 5 Solve according to the following steps: We have: $\sqrt{3^2+4^2} = 5$ then: $3sinx + 4cosx = 5 \Leftrightarrow \frac{3}{5}sinx + \frac{4}{5}cosx = 1$ Let's set: $cos\varphi = \frac{3}{5}; sin\varphi = \frac{4}{5},$ then we have: cosφ.sinx + sinφ.cosx = 1. $\Leftrightarrow sin(x+\varphi) = 1$ $\Leftrightarrow x + \varphi = \frac{\pi}{2} + k2\pi$ $\Leftrightarrow x = \frac{\pi}{2} - \varphi + k2\pi, (k \in \mathbb{Z})$ Beware of ambiguity

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		+ Specialized to discover what the real question is? Solve the same equation : $\sqrt{3}\cos x - \sin x = \sqrt{2}$
	Introduction	The mathematical symbols used when solving first-order trigonometric equations for trigonometric functions are: = ; ⇒; ⇔; ≥; <; …
Engagement	Prediction	Cyclic process
	May	For the equation: 3sinx + 4cosx = 5
	Why	Return to basic trigonometric equation form : $\sin(x + \varphi) = 1$
Verification	Checking	The condition for the equation to have a solution is: $a^2 + b^2 \geq c^2$ Because : $3^2 + 4^2 > 5^2$
	Why	Justification: $a^2 + b^2 < 0$ then the equation has no solution
	Reflection	This is a general solution for all first-order equations asinx + bcosx = c
	Expansion	Set $t = tan\frac{x}{2}$ $\Rightarrow sinx = \frac{2t}{1+t^2}; cosx = \frac{1-t^2}{1+t^2}$ The equation becomes $3.\frac{2t}{1+t^2} + 4.\frac{1-t^2}{1+t^2} = 5$ $\Leftrightarrow 6t - 4t^2 + 4 = 5.(1 + t^2)$ $\Leftrightarrow 9t^2 - 6t + 1 = 0 \quad \Leftrightarrow t = \frac{1}{3}$ Let's set: $\tan\varphi = \frac{1}{3}$ then we have $tan\frac{x}{2} = \tan\varphi$ $\Leftrightarrow x = \frac{\pi}{2} - \varphi + k2\pi, (k \in \mathbb{Z})$

Therefore, mathematical thinking within Mason's framework demonstrates that thinking is a complex process involving various steps. According to the stages, learners self-evaluate and demonstrate their activities within Mason's framework of thinking. At the same time, they can enhance their mathematical thinking abilities through this framework.

5. CONCLUSION

The paper has delved deeply into mathematical thinking and its application in solving trigonometric equations, thereby recognizing profoundly that mathematical thinking is not only a useful tool in education but also widely applicable in daily life and research.

Based on research on mathematical thinking and Mason's framework of thinking, this paper has applied them to solve trigonometric equations. The results show the effectiveness of using mathematical thinking and Mason's framework of thinking in solving complex trigonometric problems.

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Cite this Article: Ngo Tu Thanh, Nguyen Van Doc, Nguyen Minh Giam (2024). Using Mathematical Thinking in Solving Trigonometric Problems within Mason's Cognitive Framework. International Journal of Current Science Research and Review, 7(3), 1896-1903