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Integer Solutions to Sextic Equation with Five Unknowns $xy(x+y)=4(z+R)w^5$

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ABSTRACT: This paper deals with the problem of finding non-zero distinct integer solutions to the non-homogeneous sextic equation with five unknowns given by $xy(x+y)=4(z+R)w^5$.

KEY WORDS: Non-homogeneous Sextic , Sextic with five unknowns, integer solutions.

INTRODUCTION

It is well-known that a diophantine equation is an algebraic equation with integer coefficients involving two or more unknowns such that the only solutions focused are integer solutions .No doubt that diophantine equations are rich in variety [1-4] .There is no universal method available to know whether a diophantine equation has a solution or finding all solutions if it exists .For equations with more than three variables and degree at least three, very little is known. It seems that much work has not been done in solving higher degree diophantine equations. While focusing the attention on solving sextic Diophantine equations with variables at least three ,the problems illustrated in [5-24] are observed. This paper focuses on finding integer solutions to the sextic equation with four unknowns $xy(x+y)=4(z+R)w^5$.

METHOD OF ANALYSIS

The non-homogeneous Diophantine equation of degree six with four unknowns to be solved in integers is

$$xy(x+y)=4(z+R)w^5 \tag{1}$$

The process of determining non-zero distinct integer solutions to (1) are illustrated below:

Illustration 1:

Introduction of the transformations

$$x=u+v,\ y=u-v,\ z=u+2v,\ R=u-2v,\ u\neq v,2v \tag{2}$$

in (1) leads to

$$u^2-v^2=4w^5 \tag{3}$$

which is expressed as the system of double equations as below in Table 1:

Table 1: System of double equations

System	I	II	III	IV	V	VI	VII	VIII
$u+v$	$2w^5$	w^5	w^4	$2w^4$	$4w^4$	$4w^3$	$2w^3$	w^3
$u-v$	2	4	$4w$	$2w$	w	w^2	$2w^2$	$4w^2$

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Solving each of the above system of equations, the values of u, v, w are obtained.

In view of (2) , the corresponding integer solutions to (1) are exhibited below:

Solutions from System 1:

$$x = 2k^5, y = 2, z = 3k^5 - 1, R = 3 - k^5, w = k$$

Solutions from System II:

$$x = 32k^5, y = 4, z = 48k^5 - 2, R = -16k^5 + 6, w = 2k$$

Solutions from System III:

$$x = 16k^4, y = 8k, z = 24k^4 - 4k, R = -8k^4 + 12k, w = 2k$$

Solutions from System IV:

$$x = 2k^4, y = 2k, z = 3k^4 - k, R = -k^4 + 3k, w = k$$

Solutions from System V:

$$x = 64k^4, y = 2k, z = 96k^4 - k, R = -32k^4 + 3k, w = 2k$$

Solutions from System VI:

$$x = 32k^3, y = 4k^2, z = 48k^3 - 2k^2, R = -16k^3 + 6k^2, w = 2k$$

Solutions from System VII:

$$x = 2k^3, y = 2k^2, z = 3k^3 - k^2, R = -k^3 + 3k^2, w = k$$

Solutions from System VIII:

$$x = 8k^3, y = 16k^2, z = 12k^3 - 8k^2, R = -4k^3 + 24k^2, w = 2k$$

Note 1:

Taking

$$v = w^2$$

in (3) ,we get after some algebra

$$x = 2k^2(k+1)^3, y = 2k^3(k+1)^2, z = (2k+3)k^2(k+1)^2, R = (2k-1)k^2(k+1)^2,$$

$$w = k(k+1)$$

which satisfy (1).

Note 2 :

Taking

$$u = w^2$$

in (3) ,we get after some algebra

$$x = 2k^2(k+1)^3, y = -2k^3(k+1)^2, z = (4k+3)k^2(k+1)^2, R = -(4k+1)k^2(k+1)^2,$$

$$w = -k(k+1)$$

which satisfy (1).

Illustration 2 :

Introduction of the transformations

$$x = 2u + v, y = 2u - v, z = u + v, R = u - v, v \neq u, 2u \tag{4}$$

in (1) leads to

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$$4u^2 - v^2 = 2w^5 \quad (5)$$

which is expressed as the system of double equations as below in Table 2:

Table 2: System of double equations

System	I	II	III	IV	V	VI
$2u+v$	$2w^4$	$2w^3$	$2w^2$	w^4	w^3	w^2
$2u-v$	w	w^2	w^3	$2w$	$2w^2$	$2w^3$

Solving each system of double equations in Table 2 and using (4) ,the corresponding integer solutions to (1) thus obtained are exhibited below:

Solutions from System I :

$$x = 512k^4, y = 4k, z = 384k^4 - k, R = -128k^4 + 3k, w = 4k$$

Solutions from System II :

$$x = 12k^3 + k^2, y = 4k^3 + 3k^2, z = 8k^3, R = 2k^2, w = 2k$$

Solutions from System III :

$$x = 6k^2 + 2k^3, y = 2k^2 + 6k^3, z = 4k^2, R = 4k^3, w = 2k$$

Solutions from System IV :

$$x = 12k^4 + k, y = 4k^4 + 3k, z = 8k^4, R = 2k, w = 2k$$

Solutions from System V :

$$x = 8k^3, y = 8k^2, z = 6k^3 - 2k^2, R = -2k^3 + 6k^2, w = 2k$$

Solutions from System VI :

$$x = 4k^2, y = 16k^3, z = -4k^3 + k^2, R = 12k^3 - k^2, w = 2k$$

Note 3 :

Taking

$$u = w^2$$

in (5) ,we get after some algebra

$$x = 8k^2(k+2)^3, y = -8k^3(k+2)^2, z = 4(2k+3)k^2(k+2)^2,$$

$$R = -4(2k+1)k^2(k+2)^2, w = -2k(k+2)$$

which satisfy (1).

Illustration 3:

Introduction of the transformations

$$x = 2p, y = 2q, z = 2kp, R = 2kq, k \geq 1 \quad (6)$$

in (1) leads to

$$pq = kw^5 \quad (7)$$

The different values of p, q, w satisfying (7) and the corresponding integer solutions to (1) using (6) are presented in Table 3 below :

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Table 3: Integer solutions

p	q	w	$x(=2p)$	$y(=2q)$	$z(=2kp), k\geq 1$	$R(=2kq), k\geq 1$
$k\alpha^5$	1	α	$2k\alpha^5$	2	$2k^2\alpha^5$	$2k$
$k\alpha^4$	α	α	$2k\alpha^4$	2α	$2k^2\alpha^4$	$2\alpha k$
$k\alpha^3$	α^2	α	$2k\alpha^3$	$2\alpha^2$	$2k^2\alpha^3$	$2\alpha^2 k$
$k\alpha^2$	α^3	α	$2k\alpha^2$	$2\alpha^3$	$2k^2\alpha^2$	$2\alpha^3 k$
$k\alpha$	α^4	α	$2k\alpha$	$2\alpha^4$	$2k^2\alpha$	$2\alpha^4 k$
$k\alpha^5$	k^5	$k\alpha$	$2k\alpha^5$	$2k^5$	$2k^2\alpha^5$	$2k^6$
$k\alpha^4$	$k^5\alpha$	$k\alpha$	$2k\alpha^4$	$2k^5\alpha$	$2k^2\alpha^4$	$2k^6\alpha$
$k\alpha^3$	$k^5\alpha^2$	$k\alpha$	$2k\alpha^3$	$2k^5\alpha^2$	$2k^2\alpha^3$	$2k^6\alpha^2$
$k\alpha^2$	$k^5\alpha^3$	$k\alpha$	$2k\alpha^2$	$2k^5\alpha^3$	$2k^2\alpha^2$	$2k^6\alpha^3$
$k\alpha$	$k^5\alpha^4$	$k\alpha$	$2k\alpha$	$2k^5\alpha^4$	$2k^2\alpha$	$2k^6\alpha^4$

Illustration 4 :

Taking

$$x = 2zw, y = 2Rw \tag{8}$$

in (1) ,it simplifies to

$$2zR = w^2$$

which is satisfied by

$$z = 2^{2s-1}t, R = t, w = 2^s t \tag{9}$$

In view of (8) , we have

$$x = 2^{3s}t^2, y = 2^{s+1}t^2 \tag{10}$$

Thus (9) & (10) give the integer solutions to (1).

CONCLUSION

In this paper ,an attempt has been made to determine the non-zero distinct integer solutions to the non-homogeneous sextic diophantine equation with five unknowns given in the title through employing transformations. The researchers in this area may search for other choices of transformations to obtain integer solutions to the sextic diophantine equation with five unknowns under consideration.

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