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Application of Analytic Solution of Aggregate Loss Distribution through Laplace Transform on Motor Vehicle Insurance Claims Data in Indonesia

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ABSTRACT: Aggregate loss was the total loss suffered by an insured that the insurance company had to bear in one period. Aggregate losses depend on the claim frequency and claim severity. Knowing the distribution of aggregate losses was important in calculating insurance premiums. In general, there were two types of methods for determining the distribution of aggregate losses: analytical and numerical. In this research, we discussed the application of analytic solution of aggregate loss distribution through the Laplace transform. The goodness-of-fit for claim frequency data used the Chi-square test and for claim severity data, the Kolmogorov-Smirnov test was used. The data used were secondary data from the records of PT. X in 2013, consisting of data on the claim frequency and the claim severity for insured motor vehicle insurance category 3 region 25, with claims filed being partial loss. Based on the results of the application of the analytical solution of the aggregate loss distribution, it could be concluded that the claim frequency data was Poisson distributed and the claim severity data was Lindley distributed. The probability value of someone not filing a claim was 0.7316. The expected value of the aggregate loss distribution or the average claim severity for each insured approved by the insurance companies was Rp753,533.125, and it was known that the probability of an insured not making a claim exceeding Rp2,404,433.125 is 0.7316.

KEYWORDS: Aggregate loss, Chi-squared test, Kolmogorov-Smirnov test, Laplace transform, Partial loss.

INTRODUCTION

Every human being is always side by side with risk. According to Vaughan & Elliot (1978) in their book entitled "Fundamentals of Risk and Insurance", the risk is a condition in which there is a possibility of adverse deviation from the desired outcome. So it can be concluded that risk is related to the possibility of loss and uncertainty. The occurrence of risk from the insured in the field of insurance can lead to claims.

As a risk-receiving institution, insurance companies must be able to anticipate risks in the event of a claim. In risk management, insurance companies need to know the character of the risk to predict future losses. The character of the risk can be studied in a distribution model. The total loss suffered by an insured that the insurance company must bear in one period is referred to as the aggregate loss. Thus, the aggregate loss depends on the claim frequency and the claim severity.

Knowing the aggregate loss distribution is important in calculating the amount of insurance premiums. Several methods can be used to determine the aggregate loss distribution. In general, there are two types of solutions: analytical solutions and numerical solutions. Several distributions have analytical solutions in the form of the probability density function of the aggregate loss distribution through the moment generating function approach, such as the compound negative binomial-exponential distribution, any compound distribution with the claim severity is exponential distributed, and the compound Poisson distribution (Klugman et al., 2012). Mutaqin and Saidah (2021) determined the aggregate loss distribution through the numerical inverse of the characteristic function using the trapezoidal quadrature rule. When an analytical solution is not found, numerical methods such as recursive Panjer (Utami et al., 2017) and Fast Fourier Transform (Kartini et al., 2018) are used.

Sarabia et al. (2018) discussed analytical solutions to determine the aggregate loss distribution using the Laplace transform of the claim severity distribution. Sarabia et al. (2018) successfully derived analytical solutions for the compound Poisson-Lindley distribution and compound negative binomial-Lindley distribution. The results of Sarabia et al. (2018) show that there is a new and simpler method for determining the distribution of aggregate losses. The method is better than other methods because it uses the exact distribution of the data so that the results obtained are accurate, and the calculation is simpler. This study will discuss the application of the analytical solution of aggregate loss distribution through Laplace transformation on motor vehicle insurance claims data in Indonesia.

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LITERATURE REVIEW

I. Claim Frequency Distribution

Claim frequency is the number of claims made by an insured during a certain period. Distributions that are often used to model claim frequency data include the Poisson distribution and the negative binomial distribution (Omari et al., 2018).

A. Poisson distribution

The Poisson distribution is commonly used to calculate the probability of infrequent events occurring at a given interval of time (Fávero & Belfiore, 2019). A discrete random variable K is said to be Poisson distributed with parameter λ with the probability mass function:

$$P(K = k) = p_k = \frac{e^{-\lambda}\lambda^k}{k!}, \text{for } k = 0, 1, 2, \ldots$$

The expectation and variance of the Poisson distribution are as follows:

$$E(K) = \lambda$$
$$V(K) = \lambda$$

B. Negative Binomial Distribution

A discrete random variable K is said to be negative binomial distributed with parameters $r > 0$ and $0 < p < 1$ if the probability mass function is as follows (Podchishehaeva & Pankratov, 2018):

$$P(K = k) = p_k = \binom{k + r - 1}{k} p^r (1 - p)^k, \text{for } k = 0, 1, 2, \ldots$$

with the expectation and variance, respectively:

$$E(K) = \frac{r(1 - p)}{p}$$
$$V(K) = \frac{r(1 - p)}{p^2}$$

II. The Lindley Distribution as the Claim Severity Distribution

The claim severity is the amount of costs that the insurer needs to pay arising from the loss experienced by the insured. There are several distributions used to model claim severity data, including the exponential distribution or mixed exponential distribution. The distribution that will be used for modeling claim severity data in this study is the Lindley distribution.

The Lindley distribution was introduced by Lindley in 1958. Ghitany et al. (2008) stated that the Lindley distribution is a mixed distribution of the exponential distribution (θ) and the Gamma distribution $(2, \theta)$. Suppose X is a continuous random variable with Lindley distribution with parameter $\theta > 0$, then X will have the probability density function:

$$f(x) = \frac{\theta^2}{\theta + 1}(1 + x)e^{-\theta x}, \text{for } x > 0$$

The cumulative distribution function of the Lindley distribution is:

$$F(x) = 1 - \frac{e^{-\theta x}(1 + \theta + \theta x)}{\theta + 1}$$

The expectation and variance of the Lindley distribution are as follows (Zamani & Ismail, 2010):

$$E(X) = \frac{\theta + 2}{\theta(\theta + 1)}$$
$$V(X) = \frac{\theta^2 + 4\theta + 2}{\theta^2(\theta + 1)^2}$$

III. Aggregate Loss Distribution

The construction of an aggregate loss distribution is based on a collective risk model (Mwende et al., 2021):

$$S_N = X_1 + X_2 + \cdots + X_N$$

and $S_N = 0$ if $N = 0$.

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In this model, the aggregate loss S_N is calculated as the sum of individual claim severities, represented by X_j, and N is the claim frequency. The collective risk model with the X_j being independent and identically distributed random variable. There are specific independence assumptions related to this model:

1. Conditional on $N = n$, the random variables $X_1, X_2, \dots, X_n$ are independent and identically distributed.
2. Conditional on $N = n$, the distribution of the random variables $X_1, X_2, \dots, X_n$ does not depend on n.
3. Distribution of N does not depend in any values of $X_1, X_2, \dots, X_n$.

To obtain the distribution of S_N, the X_j distribution and the N distribution are modeled separately. This approach has advantages as it allows for a more accurate and flexible model. If the distribution of X_j has a tail that is significantly longer than the distribution of N, the shape of the tail in the distribution of S_N will be primarily determined by the distribution of X_j. Figure 1 provides an illustration of an aggregate loss distribution curve, which is obtained by separately modeling the X_j distribution and the N distribution.

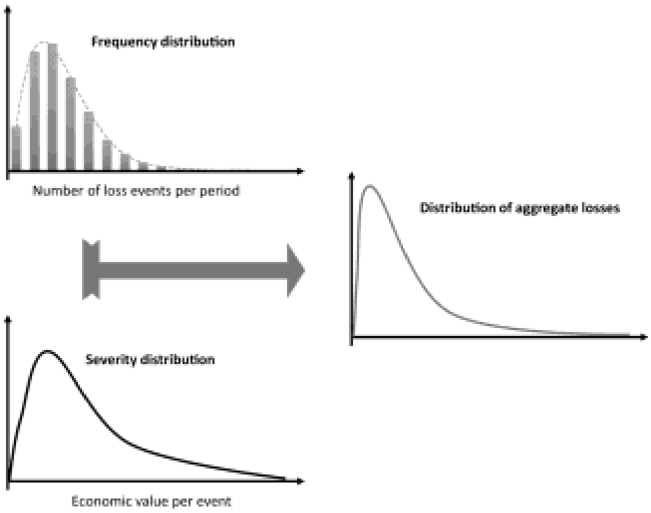

Figure 1. Aggregate loss distribution curve
Source: Bettanti & Lanati (2021)

IV. Aggregate Loss Distribution Through Laplace Transforms

Let Θ be a positive random variable with cumulative distribution function $F_\Theta(\cdot)$ and Laplace transform $\mathcal{L}_\Theta(\cdot)$. Let $f(t)$ is a function defined for $t \geq 0$ and has Laplace-Stieltjes Transform hereafter referred to as Laplace transform $\mathcal{L}\{f(t)\}$, where (Widder, 1941):

$$\mathcal{L}\{f(t)\} = E[e^{-st}] = \int_0^\infty e^{-st} f(t) dt$$

Let $X_1, X_2, \dots, X_n$ be n random samples following an exponential distribution with parameter θ, These random variables are assumed to be independent and exponentially distributed conditional on $\Theta = \theta, X_i$.

$$X_i | \Theta = \theta \sim \exp(\theta), \text{for } i = 1,2, \dots, n \quad (1)$$

$$\Theta \sim F_\Theta(.) \quad (2)$$

where $\exp(\theta)$ represents an exponential distribution with mean $\frac{1}{\theta}$.

Theorem 1: Let be Θ a positive random variable with cdf $F_\Theta(\cdot)$ and Laplace transform $\mathcal{L}_\Theta(\cdot)$. Assume that, given $\Theta = \theta$, the random variables $X_1, X_2, \dots, X_n$ are conditionally independent and distributed as exponential $\exp(\theta)$, according to model (1) – (2). Then, the pdf of the aggregated random variable $S = X_1 + X_2 + \cdots + X_n$ is given by

$$f_{S_n}(x) = \frac{x^{n-1}}{\Gamma(n)} \left\{ (-1)^n \frac{d^n}{dx^n} \mathcal{L}_\Theta(x) \right\}, \text{for } x \geq 0 \quad (3)$$

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and $f_{S_n}(x) = 0$ if $x < 0$.
Proof:
The unconditional distribution of S_n is,

$$f_{S_n}(x) = \int_0^\infty f_{S_n|\Theta}(x|\theta) d\, F_\Theta(\theta)$$

Since the conditional distribution $S_n|\Theta \sim G(n,\theta)$ is a classical gamma distribution,

$$f_{S_n}(x) = \int_0^\infty \frac{\theta^n x^{n-1} e^{-\theta x}}{\Gamma(\mathrm{n})} d\, F_\Theta(\theta)$$

$$= \frac{x^{n-1}}{\Gamma(\mathrm{n})} \int_0^\infty \theta^n e^{-\theta x} d\, F_\Theta(\theta)$$

$$= \frac{x^{n-1}}{\Gamma(\mathrm{n})} \left\{(-1)^n \frac{d^n}{dx^n} \mathcal{L}_\Theta(x)\right\}$$

Theorem 1 provides a new and simple way to obtain the probability density function of aggregate losses using the nth derivative of the Laplace transform.

If Θ follows a Lindley distribution with parameter $\theta > 0$, using equations (1) and (2) and Theorem 1, the probability function of S_n can be obtained. It is necessary to first know the Laplace transform of the Lindley distribution's probability density function, the Laplace transform of the Lindley distribution:

$$\mathcal{L}_\Theta(x) = \frac{\theta^2}{\theta + 1} \frac{\theta + x + 1}{(x+\theta)^2} \quad (4)$$

Using equation (4), the nth derivative of the Laplace transform of the Lindley distribution's probability density function can be obtained, namely:

$$\frac{d^n}{dx^n} \mathcal{L}_\Theta(x) = \frac{\theta^2}{\theta + 1} \frac{(-1)^n n!\,(\theta + x + n + 1)}{(x+\theta)^{n+2}} \quad (5)$$

By using equation (3), the probability function of S_n which result in:

$$f_{S_n}(x) = \frac{x^{n-1}}{\Gamma(\mathrm{n})} \left\{(-1)^n \frac{d^n}{dx^n} \mathcal{L}_\Theta(x)\right\}$$

$$f_{S_n}(x) = \frac{n\theta^2}{1+\theta} \frac{x^{n-1}(x+\theta+n+1)}{(x+\theta)^{n+2}}, \text{for } x > 0 \quad (6)$$

Sarabia et al. (2016) obtained several probability density functions of aggregate losses in the collective risk model. The probability density function of aggregate loss can be calculated using:

$$g_{S_N}(x) = \sum_{n=1}^\infty p_n f_{S_n}(x), \text{for } x > 0 \quad (7)$$

where p_n is the probability mass function of the claim frequency distribution and $f_{S_n}(x)$ is the probability function of S_n in equation (6). The probability density function of the aggregate loss for $x = 0$ can be calculated using:

$$g_{S_N}(0) = p_0 \quad (8)$$

where p_0 is the probability mass function of the claim frequency distribution with $n = 0$.

The random variable S_n has a cumulative distribution function:

$$\begin{aligned} G_{S_N}(x) &= \Pr(S \le x) \\ &= \sum_{n=0}^\infty p_n \Pr(S \le x\, |N = n) \\ &= g_{S_N}(0) + \int_0^x g_{S_N}(x)\, dx \end{aligned} \quad (9)$$

where $p_n = \Pr(N = n)$ is the probability mass function of N. The distribution in equation (9) is called a compound distribution.

The expectation and variance of S_N are:

$$E(S_N) = E(N)E(X) \quad (10)$$

$$V(S_N) = E(N)V(X) + V(N)\big(E(X)\big)^2 \quad (11)$$

The expected value of S_N can be used to estimate the premium that an insured needs to pay during an insurance period.

From the result of equation (11), the upper limit of the aggregate loss borne by the insurer can also be calculated using (Tse, 2009):

$$p(X) = E(S_N) + \alpha[s(S_N)] \quad (12)$$

with $\alpha \geq 0$ and $s(S_N)$ is the standard deviation of the aggregate loss distribution.

A. *Compound Poisson-Lindley Distribution*

If N is Poisson distributed with parameter $\lambda > 0$ and X is Lindley distributed with parameter $\theta > 0$, using equation (6) and p_n is the probability mass function of the Poisson distribution, the probability density function for the aggregate loss distribution for $x > 0$ is obtained:

$$g_{S_N}(x) = \frac{\theta(\theta+2) + x[2(\theta+1) + \lambda + x]}{(\theta+1)(\theta+x)^4} \lambda\theta^2 \exp\left(-\frac{\theta\lambda}{\theta+x}\right) \quad (13)$$

and using equation (8), the probability density function of the aggregate loss for $x = 0$ is:

$$g_{S_N}(0) = \exp(-\lambda) \quad (14)$$

The expected value and variance of the aggregate loss distribution can be obtained using equations (10) and (11), respectively:

$$E(S_N) = \frac{\lambda\theta + 2\lambda}{\theta(\theta+1)} \quad (15)$$

$$V(S_N) = \frac{2\lambda\theta + 6\lambda}{\theta^2(\theta+1)} \quad (16)$$

B. Compound Negative Binomial – Lindley Distribution

If N is a negative binomial distribution with parameters $r > 0$ and $0 < p < 1$ and X is a Lindley distribution with parameter $\theta > 0$, using equation (6) and p_n is the probability mass function of the negative binomial distribution, the probability density function for the aggregate loss distribution for $x > 0$ is obtained:

$$g_{S_N}(x) = \frac{\theta(\theta+2) + x[p(x+\theta-r+1) + \theta + r + 1]}{(\theta+1)(\theta+px)^{2+r}} (x+\theta)^{r-2}\theta^2 qrp^r \quad (17)$$

where $q = 1 - p$. Using equation (8), the probability density function of the aggregate loss for $x = 0$ is:

$$g_{S_N}(0) = p^r \quad (18)$$

The expected value and variance of the aggregate loss distribution can be obtained using equations (10) and (11), respectively:

$$E(S_N) = \frac{r(\theta + 2 - p\theta - 2p)}{p(\theta^2+\theta)} \quad (19)$$

$$V(S_N) = \frac{-2pr - p^2r\theta^2 - 4p^2r\theta - 2p^2r + r\theta^2 + 4r\theta + 4r}{(p\theta^2 + p\theta)^2} \quad (20)$$

METHOD

To further explain the discussion in this research, materials and methods or steps are needed to determine the distribution of aggregate losses through Laplace transform on motor vehicle insurance data in Indonesia. The data used is secondary data obtained from PT. X in 2013 for comprehensive motor vehicle insurance products during one insurance period (year) with claims use is partial loss. Data on the frequency of insured claims are presented in Table 1 and Table 2 contains the amount of the claim of the insured company of PT.X which approved by insurance companies.

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Table 1. Frequency of Insured Claim

Claim Frequency	Number of Insured
0	97
1	25
2	3
3	3
Total	128

Source: Insurance Company PT X in 2013

Table 2. Amount of Insured Claims

Insured Number	Approved claim (Rupiah)		
	1st Claim	2nd Claim	3rd Claim
1	5.170.100	1.709.350	
2	1.587.550		
3	1.975.000		
4	2.445.500		
5	1.800.000	1.900.000	6.652.100
⋮	⋮	⋮	⋮
27	1.650.000		
28	750.000		
29	1.500.000		
30	1.855.000		
31	700.000		

Source: Insurance Company PT X in 2013

Based on Table 1, it is that there were 97 insured who did not make claims during the one-year insurance period, 25 insured who made claims once during the one-year insurance period, and so on for others. Furthermore, Table 2 contains the amount of the claim of the insured company of PT. XYZ which is approved by insurance companies. For the number one insured, the amount of the 1st claim is Rp5,170,100 and the 2nd claim is Rp1,709,350 during the one-year insurance period and so on for the others.

The steps taken are as follows:

1. The goodness-of-fit test for claim frequency distribution using the Chi-square test.
2. The goodness-of-fit test for claim severity distribution using the Kolmogorov-Smirnov test.
3. Calculate the aggregate loss distribution density function for $x = 0$. If the claim frequency is Poisson distributed, the calculation uses equation (14), while if the claim frequency is negative binomial distributed, the calculation uses equation (18).
4. Calculate the aggregate loss distribution density function for $x > 0$. If the claim frequency is Poisson distributed, the calculation uses equation (13), while if the claim frequency is negative binomial distributed, the calculation uses equation (17).
5. Create an aggregate loss distribution density function curve based on the aggregate loss distribution density function that has been obtained.
6. Calculate the expected value and variance of the aggregate loss distribution. If the claim frequency is Poisson distributed, the calculation uses equations (15) and (16), while if the claim frequency is negative binomial distributed, the calculation uses equations (19) and (20).
7. Calculate the upper limit of the aggregate loss borne by the insurer, the calculation uses equation (12).

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RESULTS AND DISCUSSION

I. Goodness of Fit Tests of Poisson Distribution as Claim Frequency Distribution

The Poisson distribution is the first distribution to be tested for goodness-of-fit test because the phenomenon of insured claims is a Poisson phenomenon, namely the probability of someone making a claim is small, while the number of insured is large. The estimated value of the Poisson distribution parameter is $\hat{\boldsymbol{\lambda}} = \mathbf{0.3125}$. In Table 3 column (5) contains the values needed to calculate the Chi-square test statistic, in the last row there is the Chi-square test statistic which is 1. With a significance level $\boldsymbol{\alpha} = \mathbf{5}\%$, the quantile value of the chi-square distribution with a degree of freedom of $\mathbf{1}$ is $\mathbf{3.8415}$. It can be seen that the test statistic is smaller than the quantile value of the chi-square distribution ($\mathbf{1} < \mathbf{3.8415}$), so the null hypothesis is accepted and it can be concluded that the data on the frequency of claims of insured motor vehicle insurance PT. X in 2013 comes from a Poisson-distributed population.

Table 3. Values Required for Calculation of Chi-Square Test Statistics

Frequency of Claim (k)	Number of Insured (O_k)	Probability of Claim (p_k)	Expected Value of Claim (E_k)	$\frac{(O_k - E_k)^2}{E_k}$
(1)	(2)	(3)	(4)	(5)
0	97	0.7316	93.6468	0.1201
1	25	0.2286	29.2646	0.6215
≥2	6	0.0394	5.0489	0.1792
Total	128	1	128	1

Since it is known that the data on the frequency of insured claims for motor vehicle insurance of PT X in 2013 comes from a Poisson-distributed population, it is not necessary to do the goodness-of-fit test of the negative binomial distribution.

II. Goodness of Fit Tests of the Lindley Distribution as the Claim Severity Distribution

The goodness-of-fit test of the Lindley distribution on claim severity data of PT X motor vehicle insurance insured in 2013 using the Kolmogorov-Smirnov test. The estimated value of the parameter θ of the Lindley distribution obtained based on the Newton-Raphson iteration is θ=0.0000008294.

The results of calculations with the help of Microsoft Excel software are presented in Table 4, column (1) is the order of the data, column (2) contains the amount of insured claims data that have been sorted from the smallest value to the largest value, column (3) is the empirical distribution function value of the amount of insured claims data, column (4) is the cumulative distribution function value of the Lindley distribution, and column (5) is the absolute value of the subtraction of the empirical distribution function value and the cumulative distribution function value of the Lindley distribution.

Based on the results in Table 4, the Kolmogorov-Smirnov test statistic value can be obtained, namely:

$$D = \max_{1 \le i \le n} |F_n(x_i) - F^*(x_i)| = 0.1734$$

with a significance level $\alpha = 5\%$ and $n = 40$, the critical value based on the Kolmogorov-Smirnov test critical value table is 0.21. It can be seen that the test statistic is smaller than the critical value ($0{,}1734 < 0.21$), so the null hypothesis is accepted and it can be concluded that the data on the claim severity data of motor vehicle insurance of PT. X in 2013 comes from a Lindley-distributed population.

Table 4. Calculation Results of Kolmogorov-Smirnov Test on Claim Severity Data

i	x	$F_n(x_i)$	$F^*(x_i)$	$\lvert F_n(x_i) - F^*(x_i)\rvert$
(1)	(2)	(3)	(4)	(5)
1	50,000	0.025	0.00084	0.02416
2	97,000	0.05	0.00307	0.04693
3	125,000	0.075	0.00502	0.06998

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4	350,000	0.1	0.03481	0.06519	
5	700,000	0.125	0.11556	0.00944	
⋮	⋮	⋮	⋮		⋮
36	4,850,000	0.9	0.84354	0.05646	
37	5,170,100	0.925	0.87298	0.05202	
38	6,652,100	0.95	0.91007	0.03993	
39	6,655,000	0.975	0.9274	0.0476	
40	9,245,387	1	0.97382	0.02618	

Figure 2 displays the empirical cumulative distribution function curve and the cumulative distribution function of the Lindley distribution for the claim severity data of motor vehicle insurance claims of PT. X in 2013.

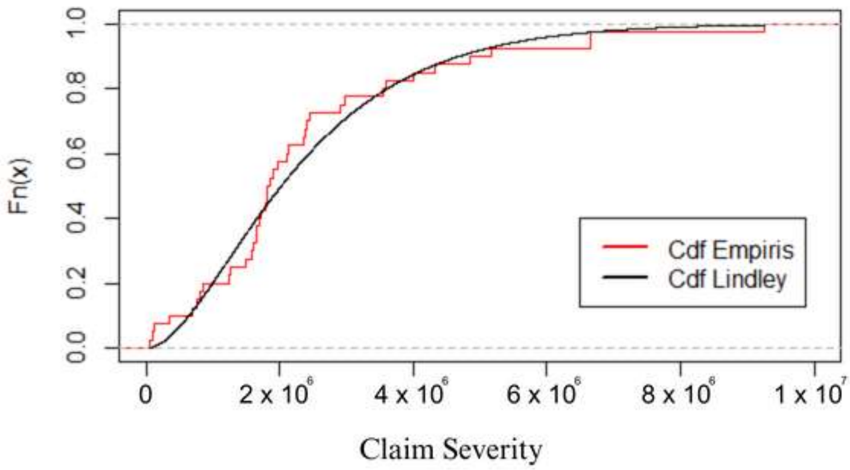

Figure 2. Empirical Distribution Function Curve and Cumulative Distribution Function of Lindley Distribution

Based on Figure 2, it can be seen that the empirical distribution function curve is close to the cumulative distribution function of the Lindley distribution for the motor vehicle insurance claim data PT. X in 2013.

III. AGGREGATE LOSS DISTRIBUTION

Based on testing the goodness-of-fit of the distribution of claim frequency and claim severity that has been carried out, it is known that the claim frequency data of motor vehicle insurance claims of PT. X in 2013 is Poisson distributed and the claim severity data of motor vehicle insurance claims of PT. X in 2013 is Lindley distributed. The estimated density function for the aggregate loss of motor vehicle insurance of PT.X in 2013 with a mixed Poisson-Lindley distribution for $x = 0$ can be calculated using equation (14), which is as follows:

$$g_{S_N}(0) = \exp(-0.3125)$$

and for $x > 0$ can be calculated using equation (13):

$$g_{S_N}(x) = \frac{0.0000008294(0.0000008294 + 2) + x[2(0.0000008294 + 1) + 0,3125 + x]}{(0.0000008294 + 1)(0.0000008294 + x)^4}(0.3125)(0.0000008294)^2 \exp\left(-\frac{(0.0000008294)(0.3125)}{0.0000008294 + x}\right)$$

Next, curve will be created for the aggregate loss distribution density function. Figure 3 presents the density function curve of the aggregate loss distribution of PT X in 2013 for the value of the aggregate loss at $x = 0$, it is known that the estimated value of the density function for the aggregate loss distribution is 0.7316, which means the probability value of someone not filing a claim is 0.7316. Furthermore, Figure 4 presents the aggregate loss distribution density function curve of PT. X in 2013 for non-zero aggregate loss values. It can be seen that the greater the value of the severity claim, the smaller the value of the aggregate loss distribution density function.

Figure 3. Density Function Curve of Aggregate Loss Distribution

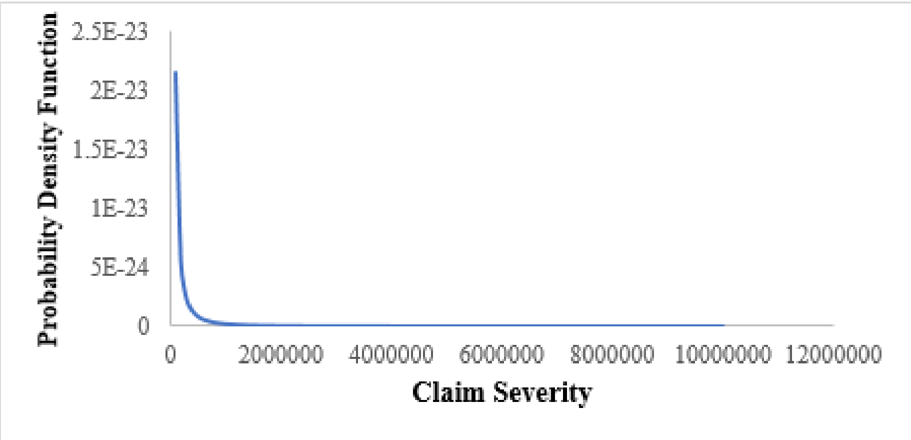

Figure 4. Non-zero Aggregate Loss Distribution Density Function Curve

The next step is to calculate the expected value and variance of the aggregate loss. Based on equation (15), the expected value of the Poisson-Lindley mixed distribution aggregate loss can be obtained, namely:

$$E(S_N) = \frac{\lambda\theta + 2\lambda}{\theta(\theta + 1)} = 753533.125$$

The expected value of the aggregate loss distribution is 753,533.125, which means that the average claim severity for each insured approved by the insurance companies is Rp753,533.125. This value can be used to estimate the amount of premium that needs to be paid by the insured during one insurance period (year).

Based on equation (16), the variance value of the aggregate loss distribution can be obtained, namely:

$$V(S_N) = \frac{2\lambda\theta + 6\lambda}{\theta^2(\theta + 1)} = 2.7255 \times 10^{12}$$

obtained the variance value of the aggregate loss distribution of 2,725,500,000,000, then the standard deviation value of the aggregate loss distribution can be obtained, namely:

$$s(S_N) = \sqrt{V(S_N)} = 1.6509 \times 10^6$$

The standard deviation value of the aggregate loss distribution is 1,650,900, this value determines the distribution of the data and shows how close the data points are to the average value.

From the standard deviation value, the upper limit of the total aggregate loss can be obtained using equation (12) with a value of $\alpha = 1$, which is as follows:

$$p(X) = E(S_N) + \alpha[s(S_N)] = 753533{,}125 + (1)(1{,}6509 \times 10^6)$$
$$= 2{,}404{,}433.125$$

then with equation (9), the cumulative distribution function $x \leq 2{,}404{,}433.125$ is obtained as follows:

$$G_{S_N}(2404433.125) = g_{S_N}(0) + \int_0^{2404433.125} g_{S_N}(2404433.125)\, dx$$
$$= 0{,}7316 + 2.465 \times 10^{-16} = 0.7316$$

then the area under the curve for $x \leq 2{,}404{,}433.125$ is 0.7316. It is known that the probability of an insured not making a claim exceeding Rp2,404,433.125 is 0.7316.

CONCLUSIONS AND SUGGESTIONS

In this research, an analytic solution has been proposed to determine the aggregate loss distribution through Laplace transform. Based on the results of the application of the analytical solution of the aggregate loss distribution on PT. X motor vehicle insurance claims data in 2013, it can be concluded that the claim frequency data comes from a Poisson-distributed population and the claim severity data comes from a Lindley-distributed population. The probability value of someone not filing a claim or the insurer not experiencing a loss is 0.7316. The expected value of the aggregate loss distribution is 753,533.125. This value can be used to estimate the amount of premium that needs to be paid by the insured during one insurance period (year). The variance value of the aggregate loss distribution is 2,725,500,000,000, and the standard deviation value of the aggregate loss distribution is 1,650,900, and the probability of an insured not making a claim exceeding Rp2,404,433.125 is 0.7316.

Based on the research that has been done, suggestions that can be given are: (1) Insurance companies are advised to consider an analytical solution through the Laplace transform in determining the distribution of aggregate losses. (2) For future researchers, it is suggested to examine analytical solutions through the Laplace transform for mixed distributions other than the Poisson-Lindley mixed distribution and the negative binomial-Lindley mixed distribution.

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