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Avoiding Ambiguous Framework in Distance-based Formation Control Using Virtual Followers

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ABSTRACT: In this study, we develop a shape formation controller based on regular distance-based control law. As distance rigidity theory may not be successful in achieving desired shape when the control system defined under minimally rigid graph. As a matter of fact, the formation may converge to ambiguous framework when the group of mobile robots did not initialize in proper way. This is a well-known issue with distance-based formation control while there are multiple equilibrium points in the dynamics. We introduce a new controller by adding virtual constraints to the system. The performance of control system is exemplified through numerical simulations and the convergence to the desired formation for all initial conditions validated with stability analysis.

KEYWORDS: Formation control, Multi-agent systems, Networked systems.

INTRODUCTION

Formation control and its application receive a lot of attentions in last decade (Aryankia and Selmic 2022) (Babazadeh, R. and Selmic, 2022) (Mehdifar et al., 2021) (Habibian and Losey, 2022). One of the basic tasks of formation control is known as acquiring a desired shape by a team of mobile robots (de Queiroz et al. , 2019), (Gazi and Passino, 2011), (Mesbahi and Egerstedt, 2010). A well-known solution to accomplish this task is to put distance constraints between a set of agents prescribed by the desired shape (de Queiroz et al., 2019), (Krick et al., 2009). This approach, known as distance-based formation control (Oh et al., 2015), with a great advantage of being implemented in decentralized manner. However, under some conditions team of mobile robots may converge to ambiguous shape instead of desired pattern and a unique position and/or orientation cannot be defined. This problem occurs due to use of minimum distance constraint for designing the controller and that will result in multiple equilibrium points in the system. To fix this problem, different techniques introduced to the challenge. By adding more distance constraint such that the formation graph is rigid (de Queiroz et al., 2019). This will remove the equilibrium points to flipped or reflected versions of the desired shape. Then, initial conditions determine whether the formation converges to the desired formation or to a ambiguous formation. Based on that, we can guarantee local stability of distance-based controllers using rigid graph. In recent years, the other methods have one thing in common. All these recent methods use additional controlled variable (or constraint) that helps to avoid formation ambiguities. For example, (Ferreira-Vazquez et al., 2016), (Jing and Wang, 2020), (Liu et al., 2020), (Liu and de Queiroz, 2008), (Park et al., 2017) beside controlling inter-agent distance, they added angle-based constraints to increase the region of attraction to the desired formation. In (Anderson et al., 2017) , the authors introduced the signed area of a triangle as an additional controlled variable and later the concept was generalized in (Liu et al., 2020, 2021) to triangulated formation graphs. There has been also similar works studied by (Cao et al., 2019), (Sugie et al., 2018). For the 3D scenarios, formations constrained with the signed volume of a tetrahedron as was represented in (Ferreira-Vazquez et al., 2016), (Lan et al., 2018). Some conditions are needed to be satisfied within all of the mentioned results such as triangulations/tetrahedralizations of the desired formation, number of agents, and/or control gains. In (Liu et al., 2020), the authors decomposed the feedback variables and control inputs into orthogonal subspaces and they called the method called the orthogonal basis approach. The same approach generalized for 3D cases (Liu and de Queiroz, 2021). In these studies, almost-global or global asymptotic stability was guaranteed with simulations and experiments that formation will converge to the desired shape without any conditions. In (Sahebsara and de Queiroz, 2021), the main cause of ambiguity was studied and then they provided a simple solution that only works based on the regular distance-based control law of for planar formations (Krick et al., 2009). Real world examples where a shape formation can be used are surveillance coverage when the formation can be also growing (Anderson et al., 2008). In this study, we want to attack same formation problem and we propose a controller where the standard distance-based

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formation controller uses predefined virtual follower information to avoid ambiguous formation. The performance of proposed formation control system is evaluated via simulations on a team of ground robotic vehicles.

PRELIMINARIES AND PROBLEM DESCRIPTION

A. *Rigid Graph Theory*

An undirected graph G is represented by the pair (V, E), where $V = \{1,2, \dots, N\}$ is the set of vertices and $E = \{(i,j) | i,j \in V, i \neq j\} \subset V \times V$ is the set of undirected edges. The set of neighbors of vertex $i \in V$ is defined as $N_i(E) = \{j \in V | (i,j) \in E\}$. If $q_i \in R^2$ is the coordinate of the i^{th} vertex of a 2D graph, then a framework F is defined as the pair (G, q) where $q = [q_1, q_2, \dots, q_N] \in R^{2N}$.

Let the map $T: R^2 \to R^2$ be such that $T(x) = Rx + d$ where $R \in SO(2)$ and $d \in R^2$. A framework $F = (G, q)$ is rigid in R^2 if all of its continuous motions satisfy $q_i(t) = T(q_i)$ for all $i = 1, \dots, N$ and $\forall t \geq 0$ <cite>Asimow79,Izmestiev</cite>. A 2D rigid framework is minimally rigid if and only if $|E| = 2N - 3$ <cite>Anderson08</cite>. The edge function of a minimally rigid framework $\gamma: R^{2N} \to R^{2N-3}$ is defined as

$$\gamma(q) = [\dots, \|q_i - q_j\|, \dots], (i,j) \in E \qquad (1)$$

such that its l^{th}component, $\|q_i - q_j\|$, relates to the l^{th} edge of E^u connecting the i^{th} and j^{th} vertices. The rigidity matrix $R: R^{2n} \to R^{|E| \times 2n}$ is defined as

$$R(q) = \frac{1}{2} \frac{\partial \gamma(q)}{\partial q} \qquad (2)$$

where we have that rank $[R(q)] \leq 2n - 3$ (Asimow and Roth, 1979). A 2D framework is said to be infinitesimally rigid if and only if rank $[R(q)] = 2n - 3$ in R^2 (Anderson et al., 2008). Frameworks $(G, q) = (G, \hat{q})$ are equivalent if $\gamma(q) = \gamma(\hat{q})$, and are congruent if $\|q_i - q_j\| = \|\hat{q}_i - \hat{q}_j\|$ for all distinct vertices i and j in V (Jackson, 2007). If infinitesimally rigid frameworks (G, q) and $(G, \hat{q})$ are equivalent but not congruent, they are flip- or flex-ambiguous (Anderson et al., 2008), (de Queiroz et al., 2019). Different type of ambiguous shape might occur during a shape formation mission and all of them caused by temporary loss of edges or using minimum number of distance constraints. In this paper, we assumed flex ambiguity will not occur and our concentration will be only on flipped ambiguity. That is, we assume that once an edge is established, it is always maintained. The flip ambiguity is exemplified in Figure 1, when robot 4 can satisfy distance constraints in both locations but the one shown with dashed lines will cause flip ambiguity.

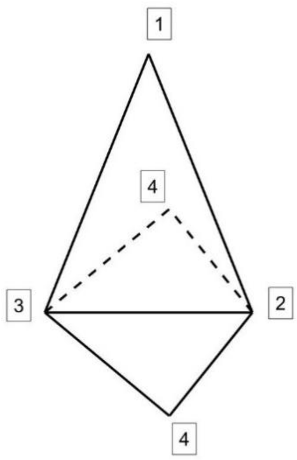

Figure 1. Flip ambiguity: edges (2,4) and (3,4) are reflected about edge (2,3) yielding equivalent frameworks.

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We also assumed the framework constructed by the Henneberg insertion of type I. This method has an interesting property that the final framework will be a minimally rigid graph. The framework begins with two vertices and an edge. Then, a vertex will be added to the graph with two edges (Bereg, 2005). Theis will result in a triangulated framework and from now on we will call such a framework, a Henneberg framework.

B. Problem Statement

A system of N mobile robots described by a set of $2N-3$ desired distances d_{ij}. The desired formation is modeled by framework $F = (G, q)$ where $G = (V, E)$, $|V| = N$, $|E| = 2N-3$, $\|q_i - q_j\| = d_{ij}, (i,j) \in E$, and $q = [q_1, \dots, q_N]$. In this system, the team of mobile robots supposed to acquire a desired 2D formation. We assumed that 1) framework F is constructed based on Henneberg insertion with an edge set $E = \{(1,2),(1,3),(2,3),\dots,(k-2,k),(k-1,k),\dots\}$ and all the mobile robots either have aligned coordinate system or known other robots' coordinate system.

The framework $F(t) = \big(G, q(t)\big)$ where $q = [q_1, \dots, q_N]$ and $q_i \in R^2$ is the position of robot i, denotes the transient model of system and F and $\check{F}$ share the same graph. When the edge $(i,j) \in E$ in F exists, that means both robots i and j can measure their relative position. Specifically, vector $q_i - q_j$ (pointing from j to i) means that robot j can sense the position of robot i relative to itself. It should be noted that motion of each robot governs based on the first-order differential equation

$$\dot{q}_i = u_i, \forall i \in V \tag{3}$$

where $u_i \in R^2$ is the control input.

We are attacking a formation problem where the robots initialized in a way that they can preserve connectivity based on their sensing range, but the formation may end up with ambiguous shape instead of the desired one. Our goal is to design a controller that removes the equilibrium points associated with flip ambiguities and the formation is only using the relative distances between robots to constrain the motion. The novelty is without imposing any additional controlled variables on the formation such as the signed area and edge angles the team of mobile robot acquire the desired shape.

C. Flip Ambiguity

Consider the formation problem, defined in previous section, that we have a minimally rigid graph under the regular distance-based controller. The final formation can converge to ambiguous framework or desired one, as illustrated in figure 2, based on how we initialize the control system. As detailed in appendix, a boundary line can be found by two agents $k-1$ and $k-2$ during the formation of framework F_k. Assume robot k should locate within this order $k, k-1$ and $k-2$ if we walk clockwise over each point, then the region separated by that boundary is the region of attraction (RoA) to desired shape $\check{F}_k(\check{q}_k)$ and the other region will be recognized as RoA to ambiguous shape $\check{F}_k(\check{q}_k^a)$. Therefore, we can claim if we initialize robot k anywhere in RoA to $\check{F}_k(\check{q}_k)$, then the final formation will satisfy all distance constraints and also form the desired shape.

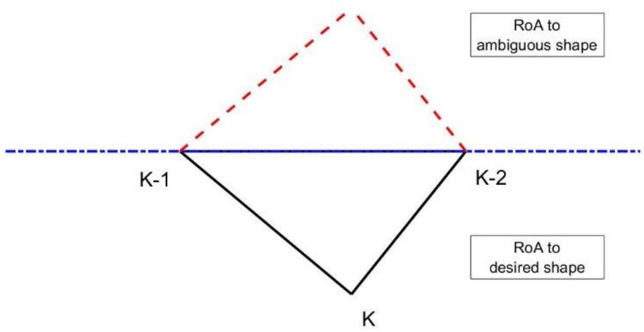

Figure 2. Regions of attraction to desired and ambiguous framework

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In the following section, we introduce a shape formation controller that guarantees the system will converge to the desired formation even if robot k initialized in the region of attraction to $\check{F}_k(\check{q}_k^a)$.

CONTROLLER DESIGN AND ANALYSIS

We present a formation controller that uses virtual agents to prevents from ambiguities in 2D shape formations by group of mobile robots regardless of what their initial conditions are. From graph rigidity theory, we know that robots connected with three undirected edges are fully constrained in planar motions and the robots with less than that requires additional constraints. In this way, as shown in figure 3, we define n virtual robots with n undirected edges connected to not fully constrained robots.

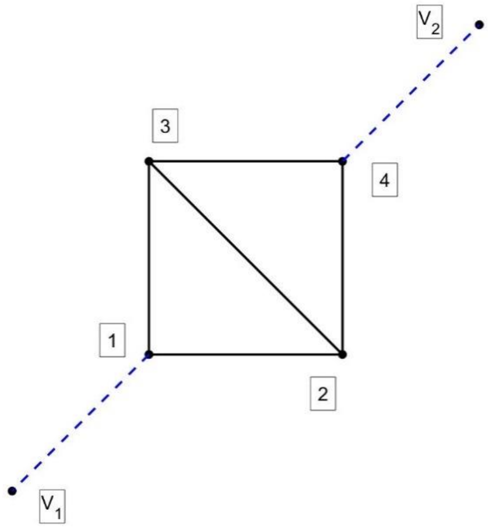

Figure 3. Assigning virtual agents to robots that may cause ambiguity

Consider the Lyapunov function candidate

$$W(e) = \frac{1}{4}\sum_{(i,j)\in E} z_{ij}^2 = \frac{1}{4}z^T z \quad (4)$$

After taking time derivative of eq, we get

$$\dot{W} = \sum_{(i,j)\in E} z_{ij}\tilde{q}_{ij}^T(u_i - u_j) \quad (5)$$

Based on eq, the distance-based formation controller is given by [cite]

$$u_i = -\alpha \sum_{j\in N_i} \tilde{q}_{ij} z_{ij} \quad (6)$$

Now consider robot k networked with the system with only two edges. This is the case when the ambiguity might occur. Therefore, if we connect an anchored virtual robot to the robot k with an undirected edge, then we can promise the ambiguity will not happen in this case. We simplify eq and eq to prove the stability of the system.

$$W_k = \frac{1}{4}\left[\left(\left\|\tilde{q}_{k(k-1)}\right\|^2 - d_{k(k-1)}^2\right)^2 + \left(\left\|\tilde{q}_{k(k-2)}\right\|^2 - d_{k(k-2)}^2\right)^2 + \left(\left\|\tilde{q}_{k(v)}\right\|^2 - d_{k(v)}^2\right)^2\right] \quad (7)$$

And

$$\dot{W}_k = \frac{1}{4}\left[\left(\left\|\tilde{q}_{k(k-1)}\right\|^2 - d_{k(k-1)}^2\right)\tilde{q}_{k(k-1)}^T + \left(\left\|\tilde{q}_{k(k-2)}\right\|^2 - d_{k(k-2)}^2\right)\tilde{q}_{k(k-2)}^T + \left(\left\|\tilde{q}_{k(v)}\right\|^2 - d_{kv}^2\right)\tilde{q}_{k(v)}^T\right] \quad (8)$$

It is obvious that there is W_k and $\dot{W}_k$ are positive definite and negative semi-definite functions, respectively and there is only one equilibrium point satisfying is $W_k = 0$ and $\dot{W}_k = 0$, which is when $\|\tilde{q}_{k(k-1)}\| = d_{k(k-1)}$, $\|\tilde{q}_{k(k-1)}\| = d_{k(k-2)}$, and $\|\tilde{q}_{k(v)}\| = d_{k(v)}$. Therefore, the control law for each robot can be presented as

$$u_k = \begin{cases} -\alpha \sum_{j\in N_i} \tilde{q}_{kj} z_{kj} + \tilde{q}_{kv} z_{kv} & , if\ size(N_k) = 2 \\ -\alpha \sum_{j\in N_i} \tilde{q}_{kj} z_{kj} & if\ size(N_k) > 2 \end{cases} \quad (9)$$

Remark: regardless of the number virtual robots introduced to the system, the final framework should be achievable by the control system and desired distances for each virtual robot should not contradict with the final desired shape.

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NUMERICAL RESULTS AND DISCUSSION

The formation control system was evaluated via simulations. The formation objective was for four robots to form a minimally rigid framework with the shape of a square (see framework with solid lines in Figure 1). To this end, N = 4 and the edge set was $V = \{(1,2),(1,3),(2,3),(2,4),(3,4)\}$ where the desired distances were $d_{12} = d_{13} = d_{24} = d_{34} = 1$ and $d_{23} = \sqrt{2}$. For the chosen 2D shape, the desired distance between robots 1 and 4, although not explicitly controlled, would be equal to $\sqrt{2}$. This distance is important because it indicates if the flip ambiguity is avoided. Two simulations were conducted with the initial conditions for robot 4 satisfying $q_4(0) \in R(\breve{q}_4^{\alpha}) \cap C(\breve{q}_3)$ and $q_4(0) \in R(\breve{q}_4^{\alpha}) \cap \bar{C}(\breve{q}_3)$. The robot trajectories from the initial time until the desired formation is acquired for each simulation are depicted in Figures 1 and 4, respectively. The corresponding distance errors, defined as $e_{ij} = \|\tilde{q}_{ij}\| - d_{ij}$, are shown in Figures 3 and 6, respectively.

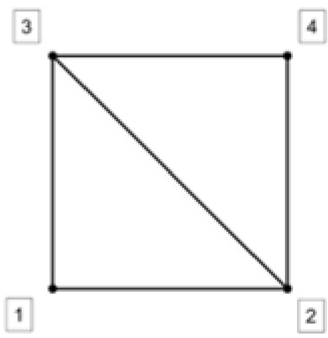

Figure 4. Desired formation F̌considered for simulations

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The Figure 5 shows robot 4 moves towards the triangle formed by robots 1, 2, and 3 and the final shape illustrates occurrence of ambiguity when we utilized the regular distance-based formation controller. This can be tracked with trajectories with blue and purple colors, and it can be seen that robot 1 and 4 converged to a common location.

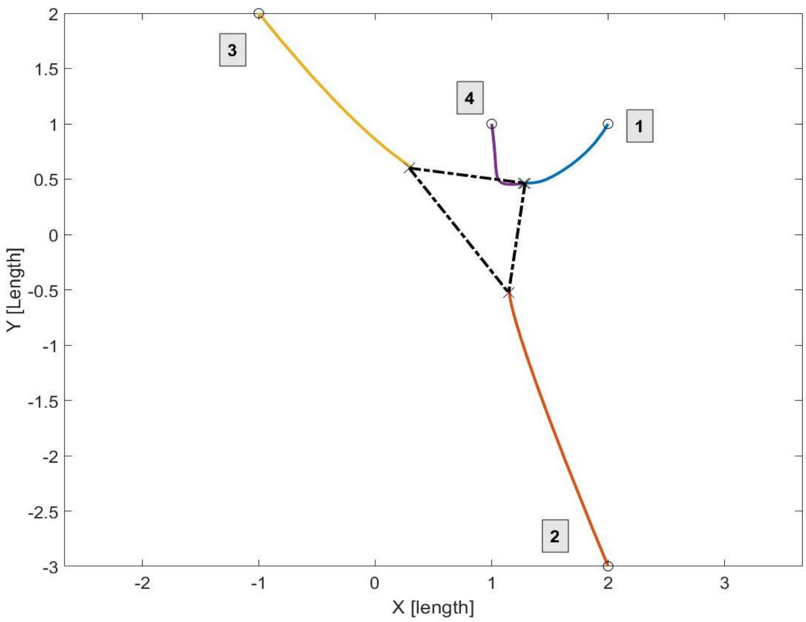

Figure 5 Robot 4 initialized in the region of attraction to ambiguous framework and inside the convex hall C.

In figure 2, we used the proposed controller (15) and with the same initial condition, the robot 1 translates all the way to acquire the desired shape. the plane formed by robots 2, 3, and 4 while the others remain immobile. The distance errors converged to zero, shown in Figure 3, to validate that the desired formation is acquired.

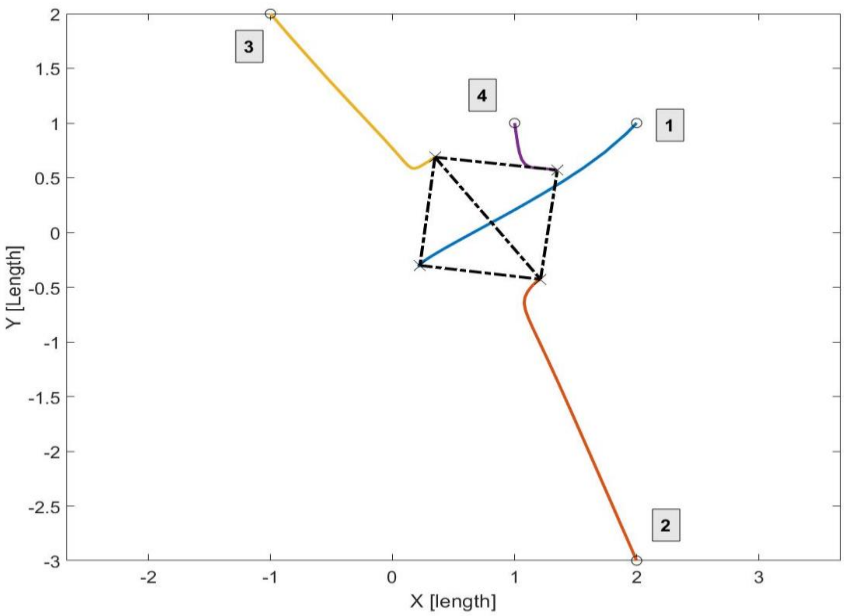

Figure 6. Robot trajectories when robot 4 initialized inside the region of attraction to ambiguity and inside the convex hull C.

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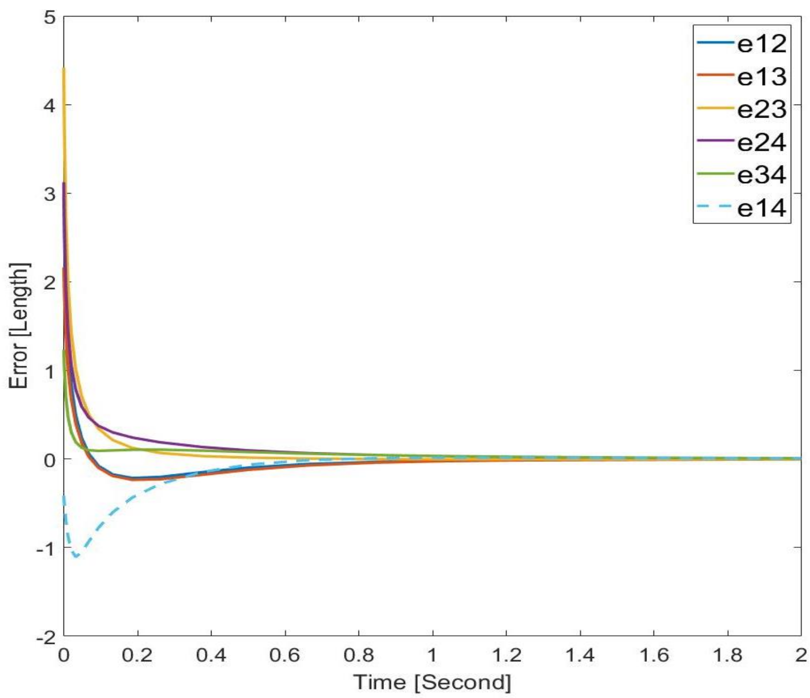

Figure 7. Distance errors when robot 4 initialized inside the region of attraction to ambiguity and inside the convex hull C.

In the second simulation, the robot 4 initiates inside the convex hull C and on the side of region of attraction to ambiguity. As shown in figure 4, after certain time robots 1 and 4 converge to ambiguous shape.

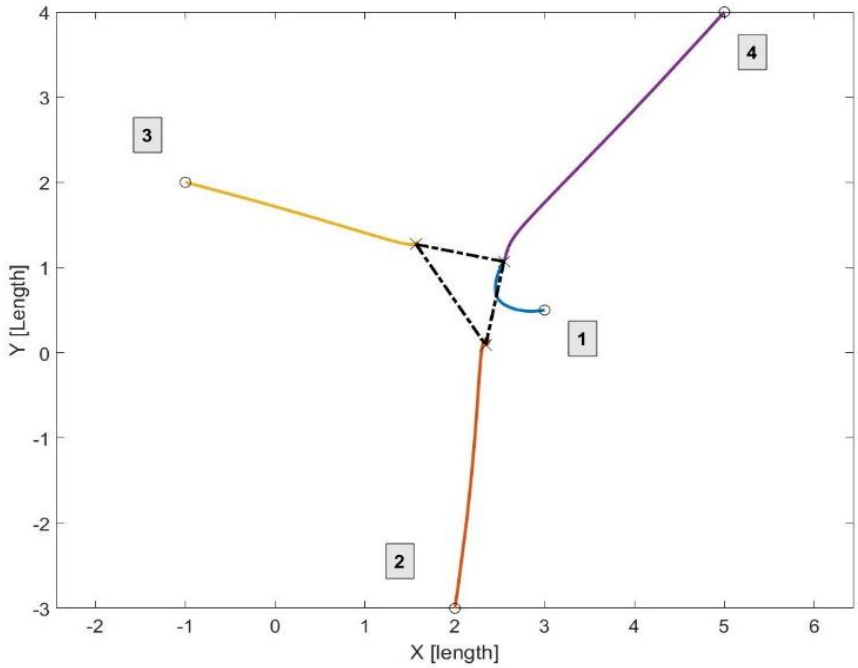

Figure 8. Robot 4 initialized in the region of attraction to ambiguous framework and outside the convex hall C.

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The performance of the proposed controller (15) with same initial condition is illustrated in figure 5. The robot 1 moves toward region of attraction to desired shape and finally acquire the desired shape. The distance errors converged to zero, shown in Figure 6, to show that the ambiguity avoided during the formation mission.

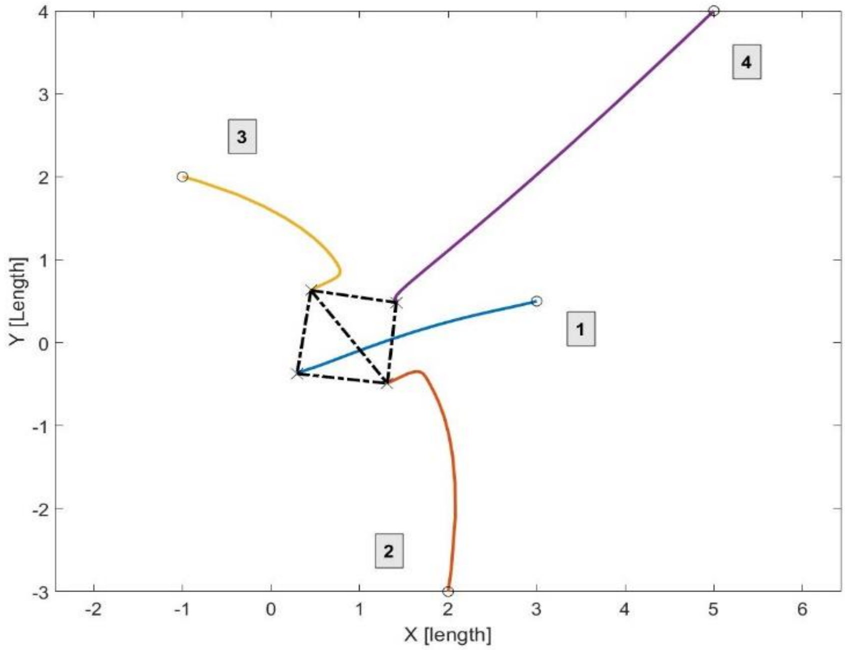

Figure 9. Robot trajectories when robot 4 initialized inside the region of attraction to ambiguity and outside the convex hull C.

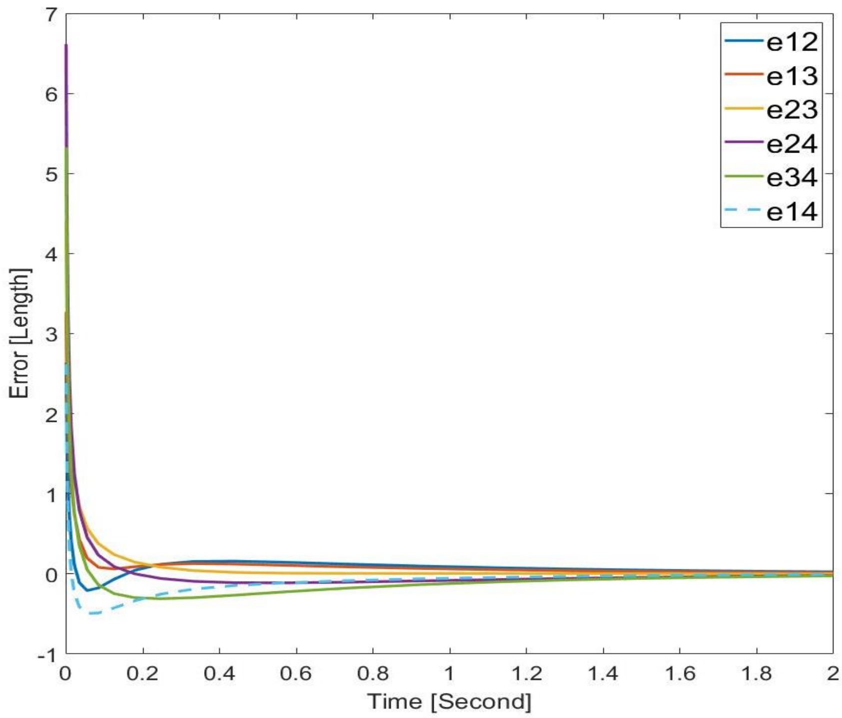

Figure 10. Distance errors when robot 4 initialized inside the region of attraction to ambiguity and outside the convex

CONCLUSIONS

In this paper, we present a controller that avoids ambiguous framework in planar distance-based formation problems. Without imposing any additional constraints, we add virtual followers, as required, to the system in a way that we obtain rigid graph, but not minimally rigid. We studied the two cases where the formation constructed using Henneberg framework. This framework will result in minimally rigid graph where there is a possibility that the formation converges to ambiguous shape. The stability of the system is studied, and the performance of controller investigated with simulations which prove that the controller was successful in preventing the flip ambiguities. In future work, we will seek to extend the proposed approach to more generalized cases and any arbitrary graphs.

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APPENDIX

We used the results in (Sahebsara and de Queiroz, 2021) to illustrate the reason behind the existence of ambiguous framework. To show this, we consider the distance-based formation controller. Let the relative position of two agents can be defined as $q_{ij} = q_i - q_j$ and then the distance error can be defined as

$$z_{ij} = \left|q_{ij}\right|^2 - {d_{ij}}^2, \forall (i,j) \in E \qquad (1)$$

Consider the Lyapunov function candidate

$$W(z) = \frac{1}{4} \sum_{(i,j)\in E} z_{ij}^2 = \frac{1}{4} z^T z \qquad (2)$$

where $e = \left[e_{12}, \dots, e_{(N-1)N}\right] \in R^{|E|}$ is the vector of errors. By taking time derivative of Lyapunov function (1), we get

$$\dot{W} = \sum_{(i,j)\in E} z_{ij} \tilde{q}_{ij}^T \left(u_i - u_j\right) \qquad (3)$$

Now we can substitute the control law (4) into the (3),

$$u_i = -\alpha \sum_{j\in N_i} \tilde{q}_{ij} z_{ij}, \forall i \in V \qquad (4)$$

where $\alpha > 0$ is a control gain. Upon substitution, we can simplify the time derivative of Lyapunov function as below

$$\dot{W} = -\alpha z^T R(q) R^T(q) z \qquad (5)$$
$$\leq -\alpha \lambda_{min}(RR^T) e^T e$$

where rigidity matrix R was used and the minimum eigenvalue of the matrix is denoted by $\lambda_{min}(.)$. The argument for $\lambda_{min}(RR^T)$ being positive is roughly that for sufficiently small initial distance errors, $z(0)$, matrix R is full row rank for all time and therefore RR^T is positive definite. Therefore, we can conclude from Lyapunov function, and its time derivative are locally exponentially stable, which implies that the formation converges to the desired formation or to a flip-ambiguous version of the desired formation when that $e = 0$. In other words, the formation converges to desired framework with the equilibrium point q and q^a denote equilibrium point ambiguous formations when $F(t) \rightarrow F(q^a)$ as $t \rightarrow \infty$. The more detail on this is provided in (Sahebsara and de Queiroz, 2021).